VE2.2: VECTORS: INTERSECTING LINES

Question

Find the point of intersection of the two lines

\[ L_1: \frac{x-10}{-2} = \frac{y+2}{2} = \frac{z-5}{1} \quad \text{and} \quad L_2: \begin{align*} x &= -2 - 2t \\ y &= -6 - 4t \\ z &= 6 + 3t \end{align*} \]

Worked Solution

\[ \text{Write } L_1 \text{ as parametric equations} \]

\[ x = 10 - 2s \quad \Rightarrow \quad x = 10 - 2s \quad \text{-- (1)} \]
\[ y = -2 + 2s \quad \Rightarrow \quad y = -2 + 2s \quad \text{-- (2)} \]
\[ z = 5 + 3s \quad \Rightarrow \quad z = 5 + 3s \quad \text{-- (3)} \]

\[ \text{NOTE: Two different lines } L_1 \text{ and } L_2 \text{ must have two different parameters } 's' \text{ and } 't'. \]

To find the intersection of these 2 lines we find a point \( P(x, y, z) \) that is common to these lines.
\[
\begin{align*}
x_1 : & \quad 10 - 2s = -2 - 2t \quad \text{(4)} \\
y_1 : & \quad -2 + 2s = -6 + 4t \quad \text{(5)} \\
z_1 : & \quad 5 + 5s = 6 + 3t \quad \text{(6)} \\
\end{align*}
\]

Solving for 't' in (4) gives \( t = -6 + s \).

Solving for 't' in (5) gives \( t = 1 + \frac{1}{2}s \).

Hence: \(-6 + s = 1 + \frac{1}{2}s \Rightarrow s = 14\).

Substituting \( s = 14 \) into:

Equation (1): \( x = 10 - 2(14) = -18 \)
Equation (2): \( y = -2 + 2(14) = 26 \)
Equation (3): \( z = 5 + 14 = 19 \)

Hence, \( P(-18, 26, 19) \) is the point of intersection of the lines \( L_1 \) and \( L_2 \).

**NOTE:**

- Equations (4) and (5) were used to solve for 't', however equating (5) and (6) could have also been used.

- Solving for 's' instead of 't' would also have resulted in the same answer.

- Check your answer by substituting \( x = -18, y = 26, z = 19 \) into \( L_1 \) and solving for 's'. The answer should be consistent, i.e., \( s = 14 \) for equations (1), (2), and (3).