VE2.1: VECTORS: EQUATION OF A LINE

Question

Find (a) parametric and (b) symmetric equations of the line $l$ through the points $P(4,1,-3)$ and $P(3,0,-1)$.

Worked Solution

To find the equation of a line, we need:

- a point $P$ on the line.
- a vector $\mathbf{u}$ parallel to the line.

Line $l$ is parallel to vector $\mathbf{u}$ which is given by:

$$\mathbf{u} = \mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$$

$$= (3,0,-1) - (4,1,-3)$$

$$\Rightarrow \mathbf{u} = (-1,-1,2)$$

(a) The Parametric Equations of a line are

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

where $\mathbf{u} = (a, b, c)$ and $P = (x_0, y_0, z_0)$ is a point on the line.

Hence, $x = 4 + (-1)t$  $y = 1 + (-1)t$  $z = -3 + 2t$
$x = 4 - t$  $y = 1 - t$  $z = -3 + 2t$
Symmetric Equations of a Line

We may eliminate \( t \) from the parametric equations

\[
x = x_0 + at \\
y = y_0 + bt \\
z = z_0 + ct
\]

to obtain the following symmetric (or Cartesian) equations of the line:

\[
\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}
\]

Since \( x = 4 - t \), \( y = 1 - t \), \( z = -3 + 2t \)

and solving for \( t \), the symmetric equations of this line are

\[
\frac{x - 4}{-1} = \frac{y - 1}{-1} = \frac{z + 3}{2}
\]

**NOTE:** If one of the numbers \( a, b \) or \( c \) is zero, we use the remaining two equations to eliminate the parameter \( t \).

Eg. if \( a = 0 \) from our question above then

\[
\frac{x - 4}{-1} = \frac{y - 1}{-1} = \frac{z + 3}{2}
\]

would be our symmetric equations of the line.