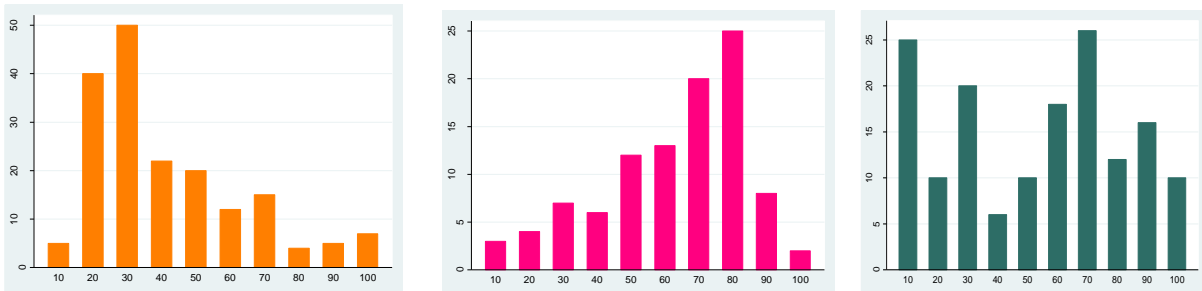


# S9. NORMAL DISTRIBUTION

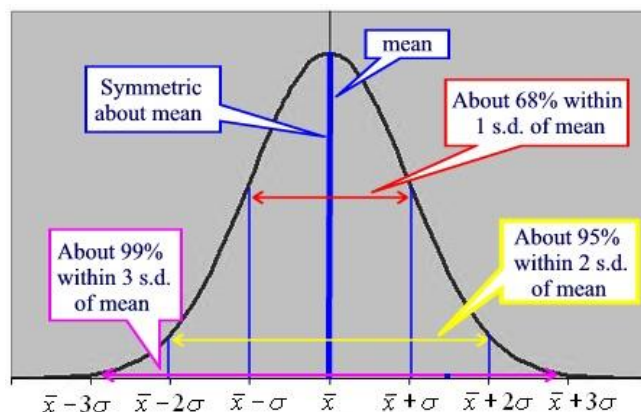
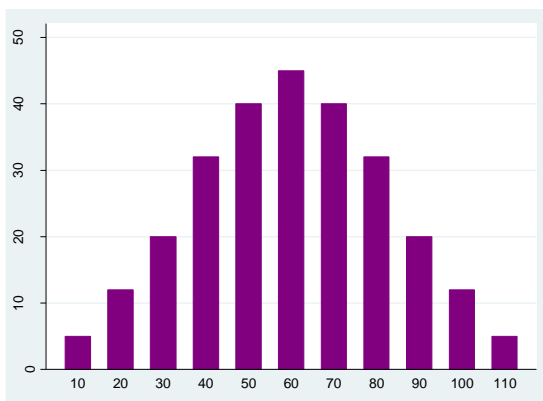
Graphical data can display different forms:



But many things that can be measured such as

- heights of people
- blood pressure
- errors in measurements
- scores on a test

follow a bell shaped curve. Such data is said to be normally distributed



Graph from Wolfram Alpha

Properties of a normal distribution

- Symmetry about the mean
- Mean = median = mode
- 50% of values greater than the mean and 50% less than the mean
- 68% of values fall within one standard deviation either side of the mean
- 95% of values fall within two standard deviations either side of the mean
- 99.7% of values fall within three standard deviation either side of the mean

*This is sometimes known as the empirical or 68-95-99 rule*

NB: Even though most of the data will fall within three standard deviations of the mean there is in theory no upper or lower bound to a normal distribution. We are just less and less likely to find values beyond these points.

## Example

If scores on an IQ test are normally distributed with mean = 100 and standard deviation = 10, what percentage of people would we expect to

- (a) score between 90 and 110?
- (b) score less than 80

- (a) Because  $90 = 100 - 10$  and  $110 = 100 + 10$  are both one standard deviation from the mean 68% of people would be expected to score between 90 and 110
- (b)  $80 = 100 - 2 \times 10$  is two standard deviations below the mean. We know that 95% of scores fall between 80 and 120 so 5% must fall outside this range. Half of these, 2.5%, will be below 80. Therefore we would expect that 2.5% of people to have IQ scores less than 80.

## Exercise

1. Scores on a general achievement test are normally distributed with a mean of 80 and a standard deviation of 15. Adam scored 95. What proportion of students had a higher score than Adam?
2. The actual weights of cereal boxes that are supposed to contain 500g are normally distributed with mean of 510g and a standard deviation of 5g. What proportion of boxes are underfilled?
3. In a maths class the bottom 16% of students are given an F grade. If the class mean is 63 and the standard deviation is 18 what score must a student get to pass?
4. If newborn birth weights in a certain hospital are normally distributed with a mean of 3200g and a standard deviation of 400g
  - (a) what percentage of babies weigh more than 3200g?
  - (b) what percentage of babies weigh between 2400g and 4000g?
  - (c) what percentage of babies weigh less than 3600g?
  - (d) if the 16% of babies with the lowest birth weights are placed in the special care nursery will a baby that weighs 2500g need special care?
5. 95% of people in a clinical study had systolic blood pressure readings between 116 and 144. If the blood pressure measurements follow a normal distribution what is the mean and standard deviation of the blood pressures for this group.
6. A class of ten students get the following marks in a test: 13, 23, 41, 55, 66, 78, 49, 33, 35, 67. If anyone who scored less than one standard deviation below the mean fails how many students will fail?

## Answers

16%   2. 2.5%   3. 45   4. (a) 50% (b) 95% (c) 84% (d) yes   5.  $\mu = 130, \sigma = 7$    6. 2