S17 TESTS OF PROPORTIONS

A test of proportion is used to determine whether or not a sample from a population represents the true proportion from the entire population.

Steps to perform a test of proportion:

1. **State the null and alternative hypotheses**

   For example, if $\hat{p}$ is the sample proportion and p the null hypothesised population proportion, then for a 2-sided test:

   H_o: $p = p_0$

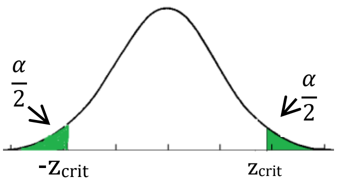
   [the proportion of the population from which the sample is drawn is the same as the given population proportion]

   H_a: $p \neq p_0$

   [the proportion of the population from which the sample is drawn is not the same as the given population proportion]

2. **A significance level α is chosen**

3. z-tables are used to find the **critical values**



4. Calculate the **test statistic**: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$

5. **A decision is made** regarding the 'reasonableness' of the test statistic if H_o is true:

   "Is the test statistic more extreme than the critical value?"
   - Yes $\Rightarrow$ Reject H_o
   - No $\Rightarrow$ Do not reject H_o

6. **State your conclusion**: There is (if you reject)/is not (if you do not reject) evidence to suggest that "……………………………………………………..."

NB: (i) Some texts and courses may use $\boldsymbol{\pi}$ *to represent the population proportion, and* $\boldsymbol{p}$ *to represent the sample proportion*

(ii) For the assumption that the test statistic is normally distributed to be valid we require $np > 5$ *and* $n(1 - p) > 5$ *(Check your course notes).*

Example

A lecturer knows that historically 60% of students do not begin their Statistics assignment until within seven days of the due date with the result that they do not do well. To address this unfortunate reality maths support services are widely promoted during the early weeks of the semester. A survey of 80 randomly selected students revealed that 42 had started the assignment before the final week. Is there evidence to suggest the promotion has been effective at the 10% significance level?

$$\hat{p} = \frac{42}{80} = 0.525$$

1. Hypotheses:

   H_o: $p = 0.6$

   [the proportion of the population from which the sample is drawn is the same as the given population proportion]

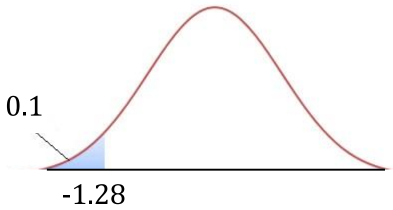
   H_a: $p < 0.6$

   [the proportion of the population from which the sample is drawn is less than the given population proportion]

2. Significance level

   Use $\alpha = 0.10$

3. Critical values



4. $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}} = \dfrac{0.525 - 0.6}{\sqrt{\dfrac{0.6(1-0.6)}{80}}} = -1.37$

5. -1.37 is more extreme than -1.28, therefore we reject H_o

6. There is evidence to suggest that the promotion has been effective and that students are beginning their assignments earlier.

NB Assumption of normality is satisfied: $np = 48 > 5$, $n(1 - p) = 32 > 5$

Exercise

1. A survey conducted ten years ago showed that 18% of students were the first in their family to attend a university. To evaluate widening participation programs a randomly selected sample of 120 enrolment records indicated that in 2017 this number had increased to 24%. Do these data provide convincing evidence to suggest that the percentage of 'first in family' students has changed over the last five years? Use $\alpha = 0.10$

2. A car manufacturer advertises that at least 80% of vehicles will obtain fuel economy of 7.0litres/100km or better. To test the claim a consumer interest group randomly selects 22 vehicles and finds that only 15 achieve this figure. Is there evidence that the companies claim is false at the 5% level of significance?

3. In a rural community hospital 6 out of 7 babies born in a particular week were boys. Assuming that in the general population the probability of female and male babies is equal, test the claim that babies born at this hospital are more likely to be boys at the 10% level of significance.

Answers

1. $z = 1.71 \Rightarrow$ reject H_o
2. $z = -1.41 \Rightarrow$ do not reject H_o
3. $z = 1.90 \Rightarrow$ reject H_o. (!) NB: Assumption of normality is NOT met