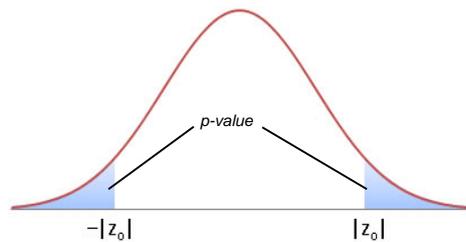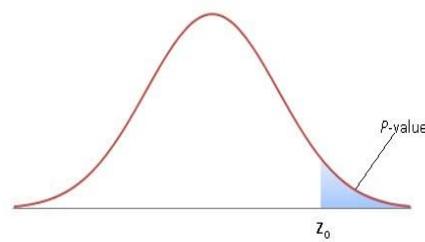# S16. P-VALUE

The p-value is a probability of obtaining a value as or more extreme than the test statistic assuming that the null hypothesis is true.

For a 2-sided test the p-value is the sum of the areas of the two regions beyond $-z_0$ and z_0:

For a 1-sided test the p-value is the area of the region beyond z_0:

NB: z_0 is the calculated value of the test statistic.

In hypothesis testing two approaches are possible when making the decision as to whether to reject the null hypothesis. So far we have compared the test statistic with the critical value(s):

"Is the test statistic more extreme than the critical value?"

$$\text{Yes} \Rightarrow \text{Reject } H_o$$
$$\text{No} \Rightarrow \text{Do not reject } H_o$$
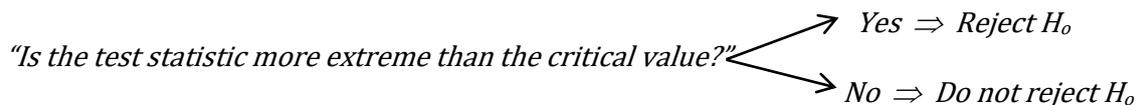
An alternative approach is to calculate the p-value of the test statistic and compare it with α.

"Is $p < $ α?"

$$\text{Yes} \Rightarrow \text{Reject } H_o$$
$$\text{No} \Rightarrow \text{Do not reject } H_o$$
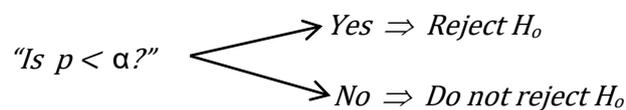
The Six Steps to conduct a hypothesis test then become:

1.   State the null and alternative hypotheses

2.   Choose the level of significance α

3.   Calculate the test statistic

4.   Compute the *p*-value of the test statistic

5.   Decide whether to reject the null hypothesis by comparing the *p*-value to α

6.   State your conclusion in words

Example:

The student queries service at a university claim that during orientation week the average wait time to be seen is 23 minutes. However this has been disputed as feedback from student focus groups indicates the wait is much longer. To test the hypothesis, the wait time for 40 randomly selected students is recorded during the next orientation week. The average wait time is found to be 25 minutes with a standard deviation of 5 minutes. Is there evidence to support the hypothesis that the average wait time exceeds 23 minutes at a 5% level of significance?

1. **Hypotheses**
   H_0: $\mu = 23$
   H_a: $\mu > 23$ [it is a one sided right tailed test as we are only interested in whether wait times are greater than 23 minutes]
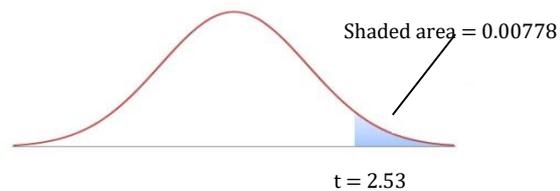
2. **Significance level**

   $\alpha = 0.05$

3. **Test statistic**

   $$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{25 - 23}{\frac{5}{\sqrt{40}}} = 2.53$$ [We are using a –t-test because the population standard deviation is unknown]

4. **p-value**

   Using a computer $p = 0.0078$:

   Shaded area = 0.00778

   t = 2.53

v	0.10	0.05	0.025	0.01	0.005	0.001
34.	1.307	1.691	2.032	2.441	2.728	3.348
35.	1.306	1.690	2.030	2.438	2.724	3.340
36.	1.306	1.688	2.028	2.434	2.719	3.333
37.	1.305	1.687	2.026	2.431	2.715	3.326
38.	1.304	1.686	2.024	2.429	2.712	3.319
39.	1.304	1.685	2.023	2.426	2.708	3.313
40.	1.303	1.684	2.021	2.423	2.704	3.307

   [Using tables we are not able to obtain an exact p-value. But we can see that with df = 39 a test statistic of 2.53 will have a p-value between 0.01 and 0.005. This is sufficient information to make a decision in step 5]

5. **Decision**
   Is $p < \alpha$? 0.00778 is less than 0.05 and therefore we reject H_o.

6. **Conclusion**
   There is evidence to suggest that the average waiting time for student queries during orientation week is more than 23 minutes

Exercise

1. A sample of 10 is drawn from a normal population and the mean is found to be 37.22. Use the p-value method to see if the sample mean is significantly less than the population mean of 40 if the population standard deviation is known to be 3.27 [use $\alpha = 0.05$] .

2. It is believed that traditionally 35% of RMIT students are aware of the SLC Maths Drop-in service. Management have recently implemented a new marketing program and results of a new survey show that out of the 100 randomly sampled students 42 knew of the service. Is there evidence to suggest that the proportion of students at RMIT who are aware of the Maths Drop-in service has changed after the implementation of the marketing initiative? Use the p-value method with $\alpha = 0.1$

3. The manufacturer of 'branbix' cereal claims the mean weight of contents of each box is 450g. A consumer interest magazine weighs a sample of 100 boxes and finds that they have a mean of 444g with a standard deviation of 28g. Use the p-value method to decide whether the sample data contradict the manufacturer's and the boxes are being under filled? [use $\alpha = 0.01$]

Answers
Your answers should be set out and contain all the steps shown above. A brief outline of the main features is given below:
1. Test statistic $z = -2.68$, $p = 0.0037$ $\Rightarrow$ reject H_o
2. Test statistic $z = \pm1.47$, $p = 0.142$ $\Rightarrow$ do not reject H_o
3. Test statistic $t = -2.14$, $p = 0.017$ $\Rightarrow$ do not reject H_o