RMIT UNIVERSITY

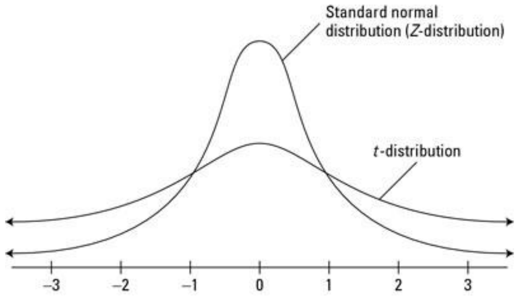
S15. T-TESTS

T-distribution

If the population standard deviation σ is unknown we may approximate with the sample standard deviation s when calculating the standard error of the distribution of means:

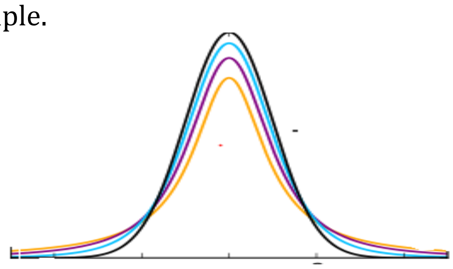
$$s_{\bar{x}} \cong \sigma_{\bar{x}} \quad \text{or} \quad \frac{s}{\sqrt{n}} \cong \frac{\sigma}{\sqrt{n}}$$

If the approximation $s \approx \sigma$ is poor (for example when $n \leq 30$), then the distribution of sample means is more closely modelled by a t-distribution.



A t-distribution has more variation in the tails than a normal distribution.

A t-distribution is always bell shaped but the shape of the curve varies according to the size of the sample.



t-distribution curves with
n = 2, 7, 16 and ∞

As the sample size increases a t-distribution becomes more like the z-distribution.

T-tests

A t-test is a hypothesis test which like the z-test is used to make decisions about the similarity of sample and population means. The test statistic is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Many students are confused about whether to use a z-test or a t-test to test their hypothesis.

Generally,

- if σ is not known, then a **t-test** is appropriate.
- if σ is known, then a **z-test** is appropriate.

However,

- Many text books only provide t-tables for $n \leq 30$. For larger samples the z-distribution is a reasonable approximation and a z-test may be used even though σ is unknown.
- If s is known to be a good approximation for σ then a z-test may be appropriate.
- Some courses provide students with more comprehensive t-tables for n up to 50 or 100.
- If a computer package such as SPSS, Minitab or EXCEL is being used for the hypothesis test then a t-test should be selected when σ is unknown.
- The test assumes sample data is drawn randomly from a normally distributed population

NB: The conventions differ between courses so check what is appropriate for your subject.

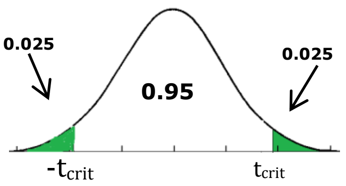
Steps in a t-test

1. **State the null and alternative hypotheses**
   For example…
   H_o: $\mu_{\bar{x}} = \mu$ [the sample mean is the same as the population mean allowing for chance variation]
   H_a: $\mu_{\bar{x}} \neq \mu$ [the sample mean is not the same as the population mean after allowing for chance variation]

   The null hypothesis is a statement that will not be rejected unless the sample data provides convincing evidence that it is false.

2. **A significance level α** is chosen [$\alpha = 0.05 \Rightarrow$ we are defining reasonable as what we can expect 95% of the time]

3. Tables or a calculator or a computer are used to find the t-value that corresponds to the chosen significance level These are called the **critical values** and depend on the degrees of freedom (n -1) and α .



4. The **test statistic** is the standardised difference between the sample mean (calculated from the given data) and the known population mean: $t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}}$

5. **A decision is made** regarding the 'reasonableness' of the test statistic if H_o is true:
   "Is the test statistic more extreme than the critical value?"
   → Yes $\Rightarrow$ Reject H_o
   → No $\Rightarrow$ Do not reject H_o

6. **State your conclusion**: There is (if you reject)/is not (if you do not reject) evidence to suggest that…

NB: (i) the decision about the null hypothesis is not made with certainty but with a level of confidence that the error in the decision is small (eg 5%).

(ii) The decision relates only to rejecting or not rejecting H_o. H_a is not mentioned in the decision, and we DO NOT accept H_o or H_a.

(iii) The steps for hypothesis testing may differ from course to course so check with your program.

Example

A manufacturer of batteries claims that on average a battery lasts 200 hours. To test this claim 7 batteries are randomly selected and this sample has an average of only 190 hours with a standard deviation of 9 hours. Does this sample provide evidence at the 1% level of significance that the manufacturers claim is incorrect?

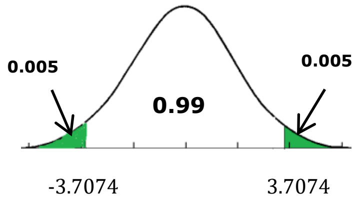
1. Hypotheses

   H_o: $\mu_{\bar{x}} = 200$ [the sample mean is the same as the population mean allowing for chance variation]

   H_a: $\mu_{\bar{x}} \neq 200$ [the sample mean is not the same as the population mean after allowing for chance variation]

2. $\alpha = 0.01$
3. $\alpha = 0.01$ and degrees of freedom = n – 1 = 6:

α (2 tail)	0.1	0.05	0.02	0.01	0.005	0.002	0.001
df							
1	6.3138	12.7065	31.8193	63.6551	127.3447	318.4930	636.0450
2	2.9200	4.3026	6.9646	9.9247	14.0887	22.3276	31.5989
3	2.3534	3.1824	4.5407	5.8408	7.4534	10.2145	12.9242
4	2.1319	2.7764	3.7470	4.6041	5.5976	7.1732	8.6103
5	2.0150	2.5706	3.3650	4.0322	4.7734	5.8934	6.8688
6	1.9432	2.4469	3.1426	3.7074	4.3168	5.2076	5.9589
7	1.8946	2.3646	2.9980	3.4995	4.0294	4.7852	5.4079
8	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0414
9	1.8331	2.2621	2.8214	3.2498	3.6896	4.2969	
10	1.8124	2.2282	2.7638	3.1693			
11	1.7959	2.2010					



   The t-distribution table shows that the critical values are -3.7074 and 3.7074. We will reject the null hypothesis if the test statistic is less than -3.7074 or greater than 3.7074

4. Test statistic: $t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}} = \dfrac{190 - 200}{\frac{9}{\sqrt{7}}} = -2.94$
5. Is -2.94 more extreme than -3.7074? No, therefore we do not reject H_o.
6. There is not evidence in this sample to suggest that the average battery life is different from 200 hours.

Exercise

1. The number of nic nac lollies in a small pack follows a normal distribution and the average is claimed to be 40. A suspicious customer buys 10 packs and counts the nic nacs in each pack. The results are: 38 45 36 40 42 44 35 41 36 40. Use a one sided test with $\alpha = 0.05$ to decide whether the customers suspicions are justified.

2. A hospital claims a new process will reduce the waiting time for surgery to treat a condition considered as non life threatening. To date the average waiting time has been 15.7 months. A random sample of 33 patients diagnosed with the condition were observed and the time till surgery recorded ($\bar{x} = 13.664, s = 2.544$). Test the hospitals claim at the 1% significance level.

3. The quality of a pre-natal program for at-risk mothers is to be assessed by comparing the weight of babies born to mothers participating in the program with the historical average of 2800g. The babies of the 25 mothers in the program had a mean birthweight of 3075g and a standard deviation of 300g. Comment on success of the program using a two sided hypothesis test and $\alpha = 0.01$.

Answers

Your answers should be set out and contain all the steps shown above. A brief outline of the main features is given below:

1. Test statistic = -0.28 $\Rightarrow$ do not reject H_o.
2. Test statistic = -4.60 $\Rightarrow$ reject H_o.
3. Test statistic = 4.58 $\Rightarrow$ reject H_o: