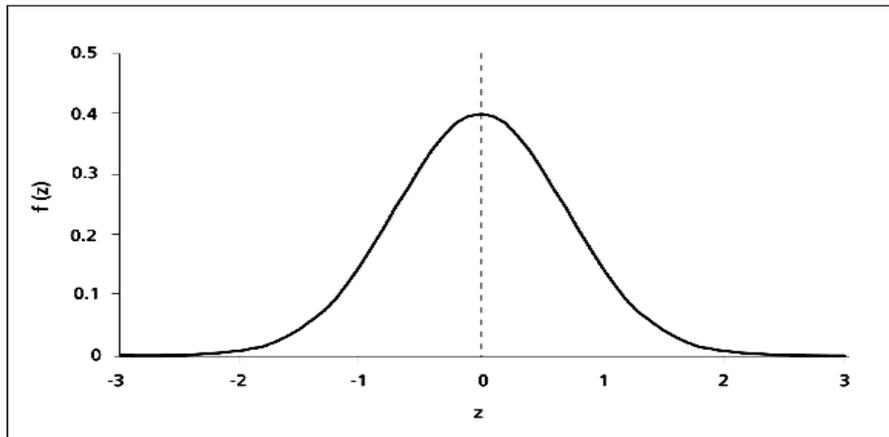


S10. STANDARD NORMAL DISTRIBUTION

The standard normal distribution (sometimes called a z-distribution) has a mean of zero ($\mu = 0$) and a standard deviation of 1 ($\sigma = 1$).



If we are working with the standard normal distribution we are not restricted to the 68-95-99 rule because tables are available to enable us to find proportions or percentages or probabilities for any value in the distribution. Tables come in different layouts, but this table gives the proportion to the left of a chosen z-value of up to 2 decimal places.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7824
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8079	0.8106

Proportion of values less than 0 is 0.5 or 50%

Proportion of values less than 0.03 is 0.512 or 51.2%

Proportion of values less than 0.75 is 0.7734 or 77.34%

We can interpret our proportions or percentages as probabilities:

$$\Pr(z < 0) = 0.5$$

$$\Pr(z < 0.03) = 0.512$$

$$\Pr(z < 0.75) = 0.7734$$

NB: A graphics calculator or a computer can also be used to find areas, proportions and probabilities in a normal distribution

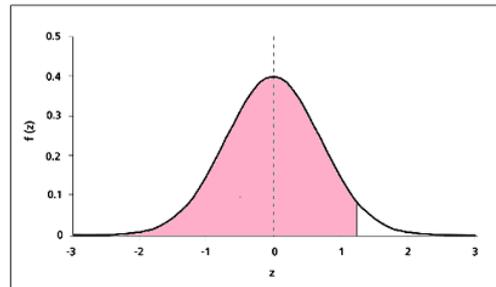
Example:

In a standard normal distribution what percentage of values will be

- (a) less than 1.28?
- (b) more than 1.28?
- (c) between 0 and 1.28?
- (d) greater than -1.28?
- (e) between -1.28 and 1.28?

(a) First draw a diagram:

We are looking for the percentage of the graph to the left of 1.28



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

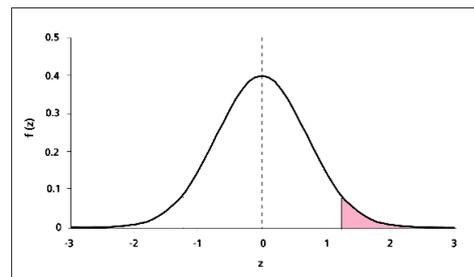
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408

We can see from the table that the percentage of values less than 1.28 is 89.97%

(b)

First draw a diagram:

We are looking for the percentage of the graph to the right of 1.28



However we cannot read areas to the right of a z-value directly from the table.

Instead we observe that

- 100% of all values lie under the curve
- The area to the left of the shaded region is the same as part (a)

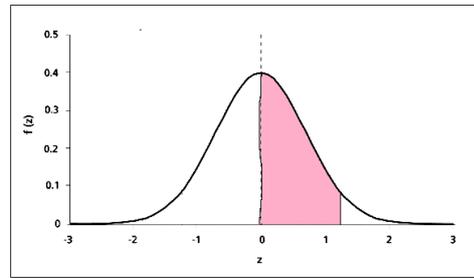
So the percentage of values to the right of 1.28 will be 100% subtract the percentage to the left of 1.28.

Therefore the percentage of values more than 1.28 is $100\% - 89.97\% = 10.03\%$

(c)

First draw a diagram:

We are looking for the percentage of the graph between 0 and 1.28



We cannot look up areas between two values directly from the table.

But we know

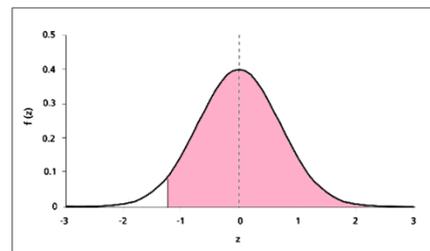
- from part (a) that 89.97% of values are less than 1.28
- 50% of values lie to the left of the mean because this is a property of our symmetrical bell curve.

Therefore the percentage of values between 0 and 1.28 is $89.97\% - 50\% = 39.97\%$

(d)

First draw a diagram:

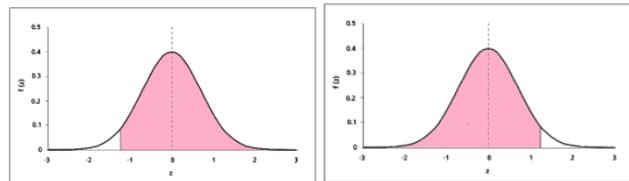
We are looking for the percentage of the graph to the right of -1.28



We cannot look up negative values in the table.

But we know

- That the bell curve is symmetrical
- The area to the right of -1.28 is the same as the area to the left of 1.28

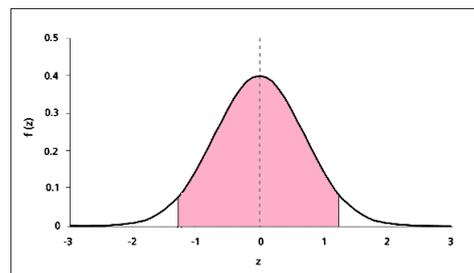


Therefore the percentage of values greater than -1.28 is 89.97%

(e)

First draw a diagram:

We are looking for the percentage of the graph between -1.28 and 1.28



To find this area we use the symmetry of the graph. Observe that the area between -1.28 and 0 is exactly the same as the area between 0 and 1.28. We know the area between 0 and 1.28 because we found it in part (c).

Therefore the percentage of the graph between -1.28 and 1.28 is $2 \times 39.97\% = 79.8\%$

Exercise

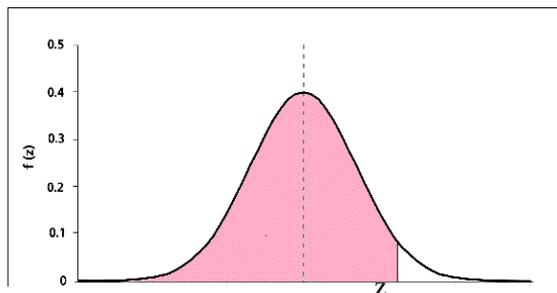
1. In a standard normal distribution what percentage of values will be

- (a) less than 1.95?
 - (b) less than -1.95?
 - (c) between -1.95 and 0?
 - (d) greater than 1.95?
 - (e) between -1.95 and 1.95
2. In a standard normal distribution what proportion of values lie between -0.5 and 1.5?
 3. In a standard normal distribution what proportion of values lie outside the interval ± 1.7 ?
 4. Given that a value in a standard normal distribution is greater than -1 what is the probability that it will be less than 2? [Hint: use conditional probability formula]

Answers

(a) 97.44% (b) 2.56% (c) 47.44% (d) 2.56% (e) 94.88% 2. 0.6247 3. 0.0892 4. 0.9729
 NB: Answers may vary slightly depending on whether calculators or tables are used

The proportion of the area under the curve to the left of a chosen value of z is given in the table below.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986