TR1.2: RIGHT-TRIANGLE TRIGONOMETRY

Trigonometry is a branch of mathematics involving the study of triangles, and has applications in fields such as engineering, surveying, navigation, optics, and electronics. Also the ability to use and manipulate trigonometric functions is necessary in other branches of mathematics, including calculus, vectors and complex numbers.

Right-angled Triangles
In a right-angled triangle the three sides are given special names.

The side opposite the right angle is called the hypotenuse (h) – this is always the longest side of the triangle.

The other two sides are named in relation to another known angle (or an unknown angle under consideration).

\[
\begin{align*}
\text{If this angle is known or under consideration} & \\
\theta & \\
\text{this side is called the opposite side because it is opposite the angle} & \\
\text{This side is called the adjacent side because it is adjacent to or near the angle} & \\
\end{align*}
\]

Trigonometric Ratios

In a right-angled triangle the following ratios are defined

\[
\begin{align*}
sine \theta & = \frac{\text{opposite side length}}{\text{hypotenuse length}} = \frac{o}{h} \\
cosine \theta & = \frac{\text{adjacent side length}}{\text{hypotenuse length}} = \frac{a}{h} \\
tangent \theta & = \frac{\text{opposite side length}}{\text{adjacent side length}} = \frac{o}{a}
\end{align*}
\]

These ratios are abbreviated to sin, cos, and tan, respectively.

A useful memory aid is Soh Cah Toa pronounced ‘so-car-tow-a’

These ratios can be used to find unknown sides and angles in right angled triangles.
Examples

1. Evaluating ratios

In the right-angled triangle below evaluate $\sin \theta$, $\cos \theta$, and $\tan \theta$.

$$\sin \theta = \frac{a}{h} = \frac{4}{5} = 0.8 \quad \cos \theta = \frac{a}{h} = \frac{3}{5} = 0.6$$

$$\tan \theta = \frac{o}{a} = \frac{4}{3} = 1.3333$$

See exercise 1

2. Finding angles.

Find the value of the angle $\theta$ in the triangle below

In this triangle we know two sides and need to find the angle $\theta$.
The known sides are the opposite side and the hypotenuse.
The ratio that relates the opposite side and the hypotenuse is the sine ratio.

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{13.4}{19.7} \quad \text{opposite side} = 13.4\text{cm}, \text{ hypotenuse} = 19.7\text{cm}$$

$$\sin \theta = 0.6082$$

This means we need the angle whose sine is 0.6082, or $\arcsin \theta$ or $\sin^{-1} 0.6082$. Using the calculator $\sin^{-1} 0.6082$ gives $\theta = 37.5$°

3. Finding side lengths

Find the value of the indicated unknown side length in each of the following right-angled triangles.

(a)

In this problem we know an angle, and the adjacent side. The side to be determined is the opposite side.
The ratio that relates these two sides is the tangent ratio.

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute in the equation opposite side = $b$, adjacent side = 42 cm, and $\theta = 27°$

$$b = 42 \times \tan 27°$$

$$b = 21.4 \text{ cm}$$
In this problem we know an angle, and the adjacent side. The unknown side is the hypotenuse. The ratio that relates these two sides is the cosine ratio.

\[
\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}
\]

Substitute in the equation adjacent side = 7, hypotenuse = \(x\), and \(\theta = 35^\circ\) 

\[
\cos 35^\circ = \frac{7}{x}
\]

\[
x = \frac{7}{\cos 35^\circ}
\]

\[
x = 8.55
\]

See Exercise 2

**Special angles and exact values**

There are some special angles that enable us to obtain exact solutions for the functions sin, cos, and tan. Exact solutions are either rational numbers or surds, not decimal approximations.

Applying the rules for sine, cosine, and tangent to the triangles below, exact values for the sine, cosine, and tangent of the angles 30°, 45°, and 60° can be found.

\[
sine = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cosine = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tanget = \frac{\text{opposite}}{\text{adjacent}}
\]

<table>
<thead>
<tr>
<th>Angle</th>
<th>Sine</th>
<th>Cosine</th>
<th>Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>(\frac{1}{2})</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{\sqrt{3}})</td>
</tr>
<tr>
<td>45°</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>(\frac{\sqrt{2}}{2})</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>(\frac{\sqrt{3}}{2})</td>
<td>(\frac{1}{2})</td>
<td>(\sqrt{3})</td>
</tr>
</tbody>
</table>
Exercises

Exercise 1
Using the right-angled triangle below find:
(a) \( \sin \theta \), (b) \( \tan \theta \), (c) \( \cos \alpha \), (d) \( \tan \alpha \) (Hint: Use Pythagoras theorem to find the hypotenuse)

![Right-angled triangle with \( \theta \) and \( a \) labeled](image)

Exercise 2
Find the value of the indicated unknown (side length or angle) in each of the following diagrams.

(a) \( a \) \( 62^\circ \) 14 cm
(b) \( \theta \) 47° 4.71 mm
(c) 4.8 cm \( z \) 6.2 cm
(d) \( x \) \( \theta \) 20.2 6.5
(e) 50° \( \alpha \) 34
(f) \( b \) 27° 42

(g) In a right angled triangle \( \sin \theta = 0.55 \) and the hypotenuse is 21mm. Find the length of each of the other two sides.

Answers
Exercise 1
(a) 12/13 = 0.9231  (b) 12/5 = 2.4  (c) 12/13 = 0.9231  (d) 5/12 = 0.4167

Exercise 2
(a) \( a = 6.6 \) cm  (b) \( a = 3.4 \) mm  (c) \( z = 7.8 \)  \( \alpha = 37.7 \)  (d) \( x = 18.8 \)  (e) \( a = 44.4 \)  (f) \( b = 47.1 \)
(g) 11.6mm. and 17.5mm.