The ‘**product rule**’ is used when we want to differentiate the product of two functions

If \( f(x) = u(x)v(x) \)

then \( f'(x) = u(x)v'(x) + u'(x)v(x) \)

which is often abbreviated to

\[
y' = uv' + u'v
\]

**Examples**

1. **Find the derivative of** \( f(x) = (x + 3)^6 (2x - 1) \)

   Let \( u = (x+3)^6 \) and let \( v = 2x - 1 \)

   \[
u' = 6(x+3)^5 \quad v' = 2
\]

   Then \( y' = uv' + u'v \)

   \[
   y' = (x + 3)^6.2 + 6(x + 3)^5(2x - 1)
   \]

   \[
   = 2(x + 3)^5(7x)
   \]

   \[
   = 14x(x + 3)^5
   \]

2. **Differentiate** \( e^x \sin2x \)

   Let \( u = e^x \) and \( v = \sin2x \)

   \[
u' = e^x \quad \text{and} \quad v' = 2\cos2x \quad \text{[using the chain rule]}
\]

   Then \( y' = uv' + u'v \)

   \[
   y' = e^x.2\cos2x + e^x \sin2x
   \]

   \[
   = 2e^x \cos2x + e^x \sin2x
   \]
Exercises

1. Use the product rule to differentiate the following
   a) $y = (x - 2)(6x + 7)$  
   b) $f(x) = (2x^2 + 4)(x^6 + 4x^2 - 2)$
   [simplify as far as possible]
   c) $y = (\sqrt{x} - 1)(x^2 + 1)$
   d) $y = (x^2 - 4x + \sqrt{x})(3x^2 + 2)$
   e) $f(x) = \sqrt{x-1}(x + 1)^2$
   f) $f(x) = (x^2 - 1)(x^2 + 1)$
   [simplify as far as possible]

2. Find the derivative of
   a) $y = e^x \tan x$
   b) $y = x^2 \log_e(x^2)$
   c) $y = \sin x \cos x$
   d) $y = x \sin x$
   e) $y = (2 - x)\tan 3x$
   f) $y = \frac{e^x}{x}$ [Hint: $\frac{1}{x} = x^{-1}$]

Answers

1. (a) $12x - 5$
   (b) $(2x^2 + 4)(5x^4 + 8x) + 4x(x^2 + 4x^2 - 2)$
   (c) $\frac{5}{2} x^\frac{3}{2} - 2x + \frac{1}{2x^\frac{1}{2}}$
   (d) $(x^3 - 4x + x^2)(12x^2) + (3x^2 - 4 + \frac{1}{2x^\frac{1}{2}})(3x^4 + 2)$
   (e) $2(x + 1)\sqrt{x-1} + \frac{(x + 1)^2}{2\sqrt{x-1}}$
   (f) $4x^3$

2. (a) $e^x \tan x + e^x \sec^2 x$
   (b) $2x + 2x \log_e(x^2)$
   (c) $\cos^2 x - \sin^2 x$
   (d) $x \cos x + \sin x$
   (e) $3(2-x)\sec^3 3x - \tan 3x$
   (f) $\frac{e^x}{x} - \frac{e^x}{x^2}$