MA1.1: INTRODUCTION TO MATRICES

A matrix is a rectangular array of elements. Matrices are usually denoted by upper case letters. The elements are usually written within brackets. The order or shape of the matrix is determined by the number of rows and columns of the matrix. The number of rows is always given first.

Example. \[ A = \begin{pmatrix} 1 & 2 & -9 \\ 2 & 5 & -3 \end{pmatrix} \]

A has two rows and 3 columns and is called a 2 x 3 matrix.

A matrix with \( m \) rows and \( n \) columns is called a matrix of order \( m \times n \).

Square matrix
A matrix with the same in which the number of rows equals the number of columns is called a square matrix.

Example \[ B = \begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix} \]

B is a square 2 x 2 matrix.

Unit Matrix
An unit (or identity) matrix is a square matrix with diagonal elements equal to one, and all other elements equal to zero. The unit matrix is usually denoted by \( I \).

\[ I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Row Matrix
A matrix with one row is called a row matrix.

Example \[ D = (2, 1, 0, 4) \] is a 1 x 4 row matrix.

Column Matrix
A matrix with one column is called a column matrix.

Example \[ E = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} \] is a 3 x 1 column matrix.
Zero Matrix
A zero matrix has all elements equal to zero. A zero matrix can be written as 0.

Example $0_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a $2 \times 2$ zero matrix

See exercise 1

Equal Matrices
For two matrices to be equal they must be the same shape and the corresponding elements must be equal.

Examples
1. If $A$ equals $B$ and $A = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 7 & 2 \\ 8 & 0 & -2 \end{pmatrix}$ then $B = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 7 & 2 \\ 8 & 0 & -2 \end{pmatrix}$

2. $C = (4, 3,)$ and $D = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ are not equal
   $C$ is not equal to $D$ because they do not have the same shape even though the numbers are the same.

3. Given: $A = \begin{pmatrix} 2 & 5 & b \\ 5 & 3 & 1 \\ 2 & 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & 7 \\ 5 & a & 1 \\ 2 & 0 & -2 \end{pmatrix}$
   If $A$ and $B$ are equal then $a = 3$ and $b = 7$

See exercise 2

Exercises
Exercise 1
1. Write down the order of the following matrices.
   (a) $\begin{pmatrix} 7 & -5 & 0 \\ 6 & 2 & -1 \end{pmatrix}$
   (b) $\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}$
   (c) $\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$
   (d) $\begin{pmatrix} 1 & 1 \\ 3 & 0 \\ -2 & 3 \end{pmatrix}$

2. (a) Write down a $2 \times 2$ identity matrix.
   (b) Write down a $3 \times 3$ zero matrix.
Exercise 2  
(a) Which of the following matrices are equal?

\[
A = \begin{pmatrix} 3 & 0 \\ 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 3 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 3 & 0 \\ 1 & -2 \end{pmatrix} \\
E = \begin{pmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \end{pmatrix} \quad F = \begin{pmatrix} 0 & 3 \end{pmatrix} \quad G = \begin{pmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \\ 1 & 3 & 0 \end{pmatrix}
\]

(b) Find \( a \) and \( b \) given that \( H = E \) (above)

\[
H = \begin{pmatrix} 3 & 5 & b \\ 2 & a & 1 \end{pmatrix}
\]

Answers  
Exercise 1  
1(a) \( 2 \times 3 \), (b) \( 2 \times 2 \)  
(c) \( 4 \times 1 \)  
(d) \( 3 \times 2 \)

2(a) \[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

(b) \[
\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

Exercise 2  
(a) \( A \) and \( D \)  
(b) \( a = 0, b = 1 \)