The **limit** of a function describes the **behaviour** of a function as the variable approaches a particular value.

**Examples**

1. **Find the limit of the function** $f(x) = x + 2$ **as** $x$ **approaches** 2

   The behaviour of $f(x)$ as $x \to 2$ is shown in the table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.95</th>
<th>1.995</th>
<th>$\to 2 \leftarrow$</th>
<th>2.005</th>
<th>2.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3.95</td>
<td>3.995</td>
<td>$\to 4 \leftarrow$</td>
<td>4.005</td>
<td>4.05</td>
</tr>
</tbody>
</table>

   The table shows that as $x$ approaches 2, $f(x)$ approaches 4.

   $\therefore \lim_{x \to 2} x + 2 = 4$

2. **Find** $\lim_{h \to 0} (5x^2 + 2xh + h)$

   The $2xh$ and $h$ terms will approach zero as $h$ approaches zero. The limit can be found by substituting zero for $h$:

   $\lim_{h \to 0} (5x^2 + 2xh + h) = 5x^2 + 2x \cdot 0 + 0$

   $= 5x^2$

3. **Find** $\lim_{h \to 0} \frac{\sin(h)}{h}$

   It is not possible to find the limit by substituting $h = 0$.

   But consider the behaviour of $f(h) = \frac{\sin(h)}{h}$ as $h \to 0$:

<table>
<thead>
<tr>
<th>$h$</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.2</th>
<th>-0.1</th>
<th>$\to 0 \leftarrow$</th>
<th>0.01</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(h)$</td>
<td>0.959</td>
<td>0.985</td>
<td>0.993</td>
<td>0.998</td>
<td>$\to ? \leftarrow$</td>
<td>0.99998</td>
<td>0.998</td>
<td>0.993</td>
<td>0.985</td>
<td>0.959</td>
</tr>
</tbody>
</table>

   It appears that $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$
Exercises

Determine the limit of the following:

1) \( \lim_{h \to 0} (4xh + 3) \)

2) \( \lim_{x \to 0} \frac{9 - x^2}{4} \)

3) \( \lim_{h \to 0} \frac{xh - 2h}{h} \)

4) \( \lim_{x \to 0} \frac{\tan(x)}{x} \)

Answers

1) 3
2) \( \frac{9}{4} \)
3) \( x - 2 \)
4) 1