FU1.5: INVERSE FUNCTIONS

Definition of an inverse function

If \( f^{-1}(x) \) is the inverse function of a one-to-one function \( f(x) \) then \( f^{-1}(x) \) is the set of ordered pairs obtained by interchanging the first and second elements in each ordered pair.

So if \((a,b) \in f \) then \((b,a) \in f^{-1} \) and if \( f(a) = b \) then \( f^{-1}(b) = a \)

The domain of \( f \) is the range of \( f^{-1} \) and the range of \( f \) is the domain of \( f^{-1} \).

For example the function \( f: \mathbb{R} \rightarrow \mathbb{R} \) defined by \( y = f(x) = \frac{x-1}{2} \) has an inverse function with rule \( f^{-1}(x) = 2x + 1 \).

So \((3,1) \) belongs to \( f \) and \((1,3) \) belongs to \( f^{-1} \), and \((-7,-4) \) belongs to \( f \) and \((-4,-7) \) belongs to \( f^{-1} \).

Graph of an inverse function

The graphs of any one-to-one function \( f \) and its inverse \( f^{-1} \) are symmetric about the line \( y = x \).

Finding an inverse function for \( y = f(x) \)

To obtain the rule for an inverse function swap the \( x \) and \( y \) coordinates in \( f \) and rearrange to express \( y \) in terms of \( x \):

Example

Find the inverse function of \( f \) where \( f(x) = 2 - 3x \)

\[
\begin{align*}
y &= 2 - 3x \\
x &= 2 - 3y & [\text{swap } x \text{ and } y] \\
x - 2 &= -3y & [\text{rearrange to make } y \text{ the subject}] \\
-x + 2 &= 3y \\
\frac{-x + 2}{3} &= y \\
\therefore \quad f^{-1}(x) &= \frac{-x + 2}{3}
\end{align*}
\]
**Exercise**

Find the inverse of each of the following one-to-one functions:

1) \( y = x + 5 \)

2) \( y = 4x \)

3) \( y = \frac{2x + 1}{3} \)

4) \( y = \sqrt{2x - 1}, \ x \geq \frac{1}{2} \)

**Answers**

1) \( f^{-1}(x) = x - 5 \)

2) \( f^{-1}(x) = \frac{x}{4} \)

3) \( f^{-1}(x) = \frac{3x - 1}{2} \)

4) \( f^{-1}(x) = \frac{x^2 + 1}{2}, \ x \geq \frac{1}{2} \)