FU1.6: INTERVAL NOTATION

Interval

Often the domain of a function will be restricted to a subset of \( \mathbb{R} \). This subset is called an interval, and the end points are \( a \) and \( b \). An interval may represented on a real number line as follows:

In inequality notation the above number line would be written as \( a \leq x \leq b \).

In interval notation the above interval would be written as \([a, b]\).

Closed Interval

Because the endpoints are included in the interval, this is called a closed interval. Square brackets are used, eg.\([2, 5]\). The end points on the real number line are represented as solid circles (or square brackets).

Open Interval

If the endpoints are excluded, the interval is an open interval. Curved brackets are used, eg \((2, 5)\).

The end points on the real number line are represented as open circles (or curved brackets).

This is written in inequality notation as \( a < x < b \). In interval notation as \((a, b)\).

Examples

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Inequality Notation</th>
<th>Line Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>([a, b])</td>
<td>(a \leq x \leq b)</td>
<td>[a, b]</td>
</tr>
<tr>
<td>((a, b))</td>
<td>(a &lt; x &lt; b)</td>
<td>((a, b))</td>
</tr>
<tr>
<td>([a, b])</td>
<td>(a \leq x &lt; b)</td>
<td>[a, b)]</td>
</tr>
</tbody>
</table>
In interval notation the smaller number is always written to the left.

i.e. \([-3, 5) \neq (5, -3]\]

Note: the symbol \(\infty\) (infinity) is not a numeral.

\(\infty\) is the concept of continuing indefinitely to the right

\(-\infty\) is the concept of continuing indefinitely to the left.

Hence we cannot write \([b, \infty]\), \([-\infty, a]\) or \(b \leq x \leq \infty\), \(-\infty \leq x \leq a\) etc..

**Examples**

1. Write the following in inequality notation and graph on a real number line.

(a) \([-2, 3)\]
   - Inequality notation: \(-2 \leq x < 3\)
   - Graph

(b) \((-\infty, 3]\)
   - Inequality notation: \(x \leq 3\)
   - Graph

2. Write the interval notation and inequality notation for the following line graphs.

(a)
   - Interval notation: \([-5, 6]\)
   - Inequality notation: \(-5 \leq x \leq 6\)

(b)
   - Interval notation: \((10, \infty)\)
   - Inequality notation: \(10 < x \leq \infty\)

See Exercise 1
Two intervals

Two (or more) subsets of \( \mathbb{R} \), with end points \( a \) and \( b \), and \( c \) and \( d \), respectively, can also be represented on a real number line.

Examples

1. 

This is written in interval notation as \([a, b] \cup [c, d]\).  
The symbol as \( \cup \) represents ‘in union with’  
In inequality notation this may be written: \( a \leq x \leq b \) with \( c \leq x \leq d \)

2. 

This is written in interval notation as \((-\infty, 2] \cup (5, 12]\).  
In inequality notation this may be written: \( x \leq 2 \) with \( 5 < x \leq 12 \)

See Exercise 2

Exercises

Exercise 1

Write the following inequalities in interval notation and graph on a real number line.  
\( (a) \ 1 \leq x \leq 10 \quad (b) \ -6 \leq x < -4 \quad (c) \ x > 5 \)

Exercise 2

Write the following in interval notation and inequality notation.  
\( (a) \quad (b) \quad (c) \)

Exercise 3

Graph the following on a real number line and write in inequality notation.

\( (a) \ (-\infty, -3] \cup (8, 13] \quad (b) \ [-1, 4] \cup [6, 9] \quad (c) \ (-\infty, 3] \cup (6, \infty) \)
Answers

Exercise 1
(a) \([1, 10]\)

(b) \([-6, -4)\)

(c) \((5, \infty)\)

Exercise 2
(a) \((-\infty, -5]\), \(x \leq -5\)
(b) \((-3, 0)\), \(-3 < x < 0\)
(c) \([-1, 4)\), \(-1 \leq x < 4\)

Exercise 3
(a) \(-\infty < x < -3\) with \(8 < x \leq 13\)

(b) \(-1 \leq x \leq 4\) with \(6 \leq x \leq 9\)

(c) \(-\infty < x \leq 3\) with \(6 < x < \infty\)