RMIT UNIVERSITY

ILS 3.2 OPERATIONS ON SURDS

Expansion of Brackets

The usual algebraic rules for expansion of brackets apply to brackets containing surds

$$a(b + c) = ab + ac$$
and
$$(a + b)(c + d) = ac + bc + ad + bd$$

Examples

1. $\sqrt{2}(\sqrt{2} + 5) = 2 + 5\sqrt{2}$

2. $2\sqrt{3}(\sqrt{3} - 3\sqrt{2}) = 2\sqrt{9} - 6\sqrt{6}$

$= 6 - 6\sqrt{6}$

3. $(7 - \sqrt{5})^2 = (7 - \sqrt{5})(7 - \sqrt{5})$

$= 49 - 7\sqrt{5 - 7\sqrt{5} + 5}$

$= 54 - 14\sqrt{5}$

4. $(\sqrt{6} - 4\sqrt{3})\left(2\sqrt{2 - 3\sqrt{5}}\right) = 2\sqrt{12} - 8\sqrt{6} - 3\sqrt{30} + 12\sqrt{15}$

$= 2\sqrt{4 \times 3} - 8\sqrt{6} - 3\sqrt{30} + 12\sqrt{15}$

$= 2 \times 2 \times \sqrt{3} - 8\sqrt{6} - 3\sqrt{30} + 12\sqrt{15}$

$= 4\sqrt{3} - 8\sqrt{6} - 3\sqrt{30} + 12\sqrt{15}$

See Exercise 1

Rationalizing surds

Sometimes fractions containing surds are required to be expressed with a *rational denominator.*

Examples

1. $\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

2. $\frac{\sqrt{5}}{3\sqrt{2}} = \frac{\sqrt{5}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{6}$

Conjugate surds

The pair of expressions $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are called *conjugate surds.* Each is the conjugate of the other.

The product of two conjugate surds does **NOT** contain any surd term!

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

Eg: $(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3}) = (\sqrt{10})^2 - (\sqrt{3})^2$

$= 10 - 3$

$= 7$

We make use of this property of conjugates to rationalize denominators of the form $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$

Example: $\frac{\sqrt{3}}{5+\sqrt{2}} = \frac{\sqrt{3}}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$

$= \frac{\sqrt{3}(5-\sqrt{2})}{25-2}$

$= \frac{5\sqrt{3}-\sqrt{6}}{23}$

See Exercise 2

Operations on fractions that contain surds

When adding and subtracting fractions containing surds it is generally advisable to first rationalize each fraction:

Example

$\frac{2}{3\sqrt{2}+1} + \frac{1}{\sqrt{3}-\sqrt{2}} = \frac{2}{3\sqrt{2}+1} \times \frac{3\sqrt{2}-1}{3\sqrt{2}-1} + \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{6\sqrt{2}-2}{18-1} + \frac{\sqrt{3}+\sqrt{2}}{3-2}$

$= \frac{6\sqrt{2}-2}{17} + \frac{\sqrt{3}+\sqrt{2}}{1}$

$= \frac{6\sqrt{2}-2}{17} + \frac{17(\sqrt{3}+\sqrt{2})}{17}$

$= \frac{6\sqrt{2}-2+17(\sqrt{3}+\sqrt{2})}{17}$

$= \frac{23\sqrt{2}-2+17\sqrt{3}}{17}$

See Exercise 3

Exercise 1

Expand the brackets and simplify if possible

(a) $\sqrt{2}(\sqrt{2}-8)$ (b) $(2+\sqrt{3})(\sqrt{3}-4)$ (c) $(1+\sqrt{10})^2$ (d) $(\sqrt{11}+3)(\sqrt{11}-3)$

Exercise 2

Express the following fractions with a rational denominator in simplest form:

1. (a) $\frac{\sqrt{5}}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{10}}$ (c) $\frac{2\sqrt{18}}{\sqrt{8}}$
2. (a) $\frac{2+\sqrt{3}}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (c) $\frac{\sqrt{3}+2}{\sqrt{3}-2}$

Exercise 3

Evaluate and express with a rational denominator $\frac{2}{\sqrt{3}-1}+\frac{3}{2-\sqrt{3}}$

Answers

Exercise 1

(a) $2-8\sqrt{2}$ (b) $-5-2\sqrt{3}$ (c) $11+2\sqrt{10}$ (d) 2

Exercise 2

1. (a) $\frac{\sqrt{10}}{2}$ (b) $\frac{\sqrt{10}}{10}$ (c) 3
2. (a) $\frac{2\sqrt{2}+\sqrt{6}}{2}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $-7-4\sqrt{3}$

Exercise 3

$4\sqrt{3}+7$