ILS 3.1 SIMPLIFYING SURDS

Definition

A ***surd*** is an irrational number resulting from a radical expression that ***cannot*** be evaluated directly.

For example $\sqrt{2}, \sqrt{3}, \frac{1}{\sqrt{8}}, \sqrt{20}, \sqrt[3]{2}, \sqrt[3]{9}, \sqrt[4]{8}$, etc are all surds,

but $\sqrt{1}, \sqrt{9}, \frac{1}{\sqrt{100}}, \sqrt[3]{8}, \sqrt[4]{16}, \sqrt[3]{64}$ are not.

Surds *cannot* be expressed exactly in the form $\frac{a}{b}$, and can only be approximated by a decimal.

eg: $\sqrt{2} \sim 1.414$

See Exercise 1

Simplifying Surds

If a surd has a square factor ie 4, 9, 16, 25, 36…, then it can be simplified.

Examples

1. $\sqrt{200} = \sqrt{100 \times 2}$

$= \sqrt{100} \times \sqrt{2}$ [NB: It is best to find the *largest* square number factor!]

$= 10\sqrt{2}$

2. $3\sqrt{48} = 3\sqrt{16 \times 3}$

$= 3\sqrt{16} \times \sqrt{3}\sqrt{3}$

$= 3 \times 4 \times \sqrt{3}$

$= 12\sqrt{3}$

3. $\frac{\sqrt[3]{24}}{4} = \frac{\sqrt[3]{8 \times 3}}{4}$

$= \frac{\sqrt[3]{8} \times \sqrt[3]{3}}{4} \; \frac{\sqrt[3]{8} \times \sqrt[3]{3}}{4}$

$= \frac{2 \times \sqrt[3]{3}}{4}$

$= \frac{\sqrt[3]{3}}{2}$

See Exercise 2

Addition and Subtraction

Only like surds can be added or subtracted

Examples

1. $8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5}$
2. $2\sqrt{3} + 5\sqrt{7} - 10\sqrt{3} = 5\sqrt{7} - 8\sqrt{3}$

Sometimes it is necessary to simplify each surd first.

3. $\sqrt{18} - \sqrt{8} - \sqrt{20} = \sqrt{9 \times 2} - \sqrt{4 \times 2} - \sqrt{4 \times 5}$

$$= 3\sqrt{2} - 2\sqrt{2} - 2\sqrt{5}$$

$$= \sqrt{2} - 2\sqrt{5}$$

NB: $\sqrt{5} + \sqrt{7} \neq \sqrt{12}$ because $\sqrt{5}$ *and* $\sqrt{7}$ are not 'like' surds.

See Exercise 3

Multiplication

To multiply surds the whole numbers are multiplied together, as are the numbers enclosed by the radical sign.

That is: $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

Examples:

1. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$
2. $3\sqrt{3} \times 4\sqrt{5} = 12\sqrt{15}$

Sometimes it is possible to simplify after multiplication

3. $2\sqrt{10} \times 7\sqrt{6} = 14\sqrt{60}$

$$= 14\sqrt{4 \times 15}$$

$$= 14 \times 2\sqrt{15}$$

$$= 28\sqrt{15}$$

See Exercise 4

Division

Expressions such as $\sqrt{a} \div \sqrt{b}$ may be expressed as fractions and simplified using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Examples

1. $\frac{\sqrt{18}}{\sqrt{6}} = \sqrt{\frac{18}{6}} = \sqrt{3}$
2. $\frac{4}{\sqrt{2}} = \frac{\sqrt{16}}{\sqrt{2}} = \sqrt{\frac{16}{2}} = \sqrt{8} = 2\sqrt{2}$

See Exercise 5

Exercise 1

Decide whether the following radical expressions are rational or irrational and evaluate exactly if rational:

(a) $\sqrt{25}$ (b) $\sqrt{2.25}$ (c) $\sqrt{\frac{1}{10}}$ (d) $\sqrt[3]{64}$ (e) $\sqrt{12 \times 18}$

Exercise 2

Simplify

(a) $\sqrt{20}$ (b) $\sqrt{48}$ (c) $\sqrt{500}$ (d) $2\sqrt{12}$ (e) $\frac{2}{\sqrt{20}}$

Exercise 3

Simplify

(a) $2\sqrt{3} + 5\sqrt{3}$ (b) $3\sqrt{7} + 5\sqrt{7} - 3\sqrt{7}$

(c) $\sqrt{2} + \sqrt{7} + 3\sqrt{2} - 4\sqrt{7}$ (d) $\sqrt{54} - \sqrt{24}$

Exercise 4

Simplify

(a) $\sqrt{3} \times \sqrt{10}$ (b) $2\sqrt{7} \times 5\sqrt{32}$

(c) $\sqrt{10} \times 3\sqrt{10}$ (d) $2\sqrt{8} \times 2\sqrt{50} \times \sqrt{2}$

Exercise 5

Simplify

(a) $\frac{\sqrt{12}}{\sqrt{6}}$ (b) $\frac{\sqrt{32}}{\sqrt{2}}$

(c) $\frac{3\sqrt{28}}{\sqrt{7}}$ (d) $\frac{4\sqrt{2} \times 3\sqrt{3}}{6\sqrt{6}}$

Answers

Exercise 1

(a) 5 rational (b) 1.5 rational (c) irrational (d) 4 rational (e) irrational

Exercise 2

(a) $2\sqrt{5}$ (b) $4\sqrt{3}$ (c) $10\sqrt{5}$ (d) $4\sqrt{3}$ (e) $\frac{1}{\sqrt{5}}$

Exercise 3

(a) $7\sqrt{3}$ (b) $5\sqrt{7}$ (c) $4\sqrt{2} - 3\sqrt{7}$ (d) $\sqrt{6}$

Exercise 4

(a) $\sqrt{30}$ (b) $10\sqrt{21}$ (c) 30 (d) $80\sqrt{2}$

Exercise 5

(a) $\sqrt{2}$ (b) 4 (c) 6 (d) 2