STUDY AND LEARNING CENTRE

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STUDY TIPS

RMIT UNIVERSITY

ILS 2.1 LOGARITHMS

Logarithms

Logarithms are closely related to indices.

The logarithm of a number is the power to which the base must be raised to give the number.

This means that a **logarithm** is an **index.**

Now, let's revise what it means.

Index or logarithm

Number ⟶ $8 = 2^3$

base

This familiar statement may be written as $\log_2 8 = 3$ Index or logarithm

base

Generally:

$$a^x = n \iff \log_a n = x$$ (follow the arrows and read $a^x = n$) $a > 0$

Examples

1. Re-write these equations in logarithmic notation:

(a) $5^3 = 125 \Rightarrow \log_5 125 = 3$ Using the above general expression:

(b) $3^{-2} = \frac{1}{9} \Rightarrow \log_3 \frac{1}{9} = -2$ Using the above general expression

(c) $25^{\frac{1}{2}} = 5 \Rightarrow \log_{25} 5 = \frac{1}{2}$ Using the above general expression

To evaluate a logarithm:

1. write the number in index form, with same base as the logarithm.
2. use the definition of a logarithm to evaluate.

Example1: Evaluate $\log_5 125$

$125 = 5^3$ 125 in index form with base 5

$\therefore \log_5 125 = 3$ equivalent logarithm statement

Example 2: Evaluate $\log_{10} 0.0001$

$0.0001 = 10^{-4}$ 0.0001 in index form with base 10

$\therefore \log_{10} 0.0001 = -4$ equivalent logarithm statement

Two important logs to remember

$a^0 = 1 \quad \Rightarrow \quad \log_a 1 = 0$

In words: **For any base, the log of 1 is zero.**

$a = a^1 \quad \Rightarrow \quad \log_a a = 1$

In words: **The log of any number with itself as base is 1.**

Logarithm Laws

Corresponding to **the three index laws**, there are **three laws of logarithms** to help in manipulating logarithms.

First Logarithm Law: $\log_a(mn) = \log_a m + \log_a n$

Second Logarithm Law: $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

Third Logarithm Law: $\log_a(m^p) = p \log_a m$

Example 1: $\log_a 15 = \log_a(5\times3) = \log_a 5 + \log_a 3$ Using the first logarithm law

Example 2: $\log_a\left(\frac{12}{5}\right) = \log_a 12 - \log_a 5$ Using the second logarithm law

Example 3: $\log_a(9^2) = 2\log_a 9 = 2\log_a(3^2) = 4\log_a 3$ Using the third logarithm law

Note: To use the logarithm laws, all logarithms must have the same base.

These laws together with the definition of a logarithm can be used to simplify and evaluate logarithmic expressions and to solve equations involving logarithms.

Examples

1. Solve: $\log_2 x = 5$

$\log_2 x = 5 \;\Rightarrow\; 2^5 = x$ *using the definition of a logarithm*

Therefore $x = 32$

2. Simplify: $\log_{10} 6 + \log_{10} 2$

$\log_{10} 6 + \log_{10} 2 = \log_{10}(6\times2)$ *using first log. law*

$= \log_{10} 12$

3. Simplify: $\log_3 6a + \log_3 b - \log_3 2a$

$$= \log_3 (6a \times b) - \log_3 2a$$

$$= \log_3 \left(\frac{6a \times b}{2a} \right)$$

$$= \log_3 3b$$

4. Simplify: $\frac{1}{2}\log_{10} 36 - \log_{10} 15 + 2\log_{10} 5$

$$= \log_{10} 36^{\frac{1}{2}} - \log_{10} 15 + \log_{10} 5^2$$

$$= \log_{10} 6 - \log_{10} 15 + \log_{10} 25$$

$$= \log_{10} \frac{6 \times 25}{15}$$

$$= \log_{10} 10$$

$$= 1$$

5. Simplify: $3\log_{10} a - 2\log_{10} b + 2\log_{10} 5$

$$= \log_{10} a^3 - \log_{10} b^2 + \log_{10} 5^2$$

$$= \log_{10} \left(\frac{25a^3}{b^2} \right)$$

6. Solve for *x*: $\log_{10}(2x+1) = \log_{10} 3$

Therefore $2x+1 = 3$

$x = 1$

Exercise 1

Write in logarithm form.

(a) $3^2 = 9$ (b) $10^4 = 10,000$ (c) $10^{-2} = 0.01$ (d) $e^a = b$

Exercise 2

Evaluate without using a calculator.

(a) $\log_7 49$ (b) $\log_{10} \sqrt{10}$ (c) $\log_5 1$ (d) $\log_{10} 100\ 000$ (e) $\log_5 5$

Exercise 3

1. Simplify.

(a) $\log_4 8 + \log_4 3 - \log_4 2$ (b) $\frac{1}{2}\log_{10} 25 - \log_{10} 4 + 2\log_{10} 3$

(c) $\log_e 2e^3 + 2\log_e 3 - \log_e 18$ (d) $\log_a 4 + 2\log_a 3 - 2\log_a 6$

(e) $\frac{1}{2}\log_{10} a^2 + 3\log_{10} b - \log_{10} ab^2$

2. Solve for x

(a) $\log_{10} x = \log_{10} 4 - \log_{10} 2$ (b) $\log_2 2x - 2\log_2 3 = \log_2 6$

Answers

Exercise 1

(a) $\log_3 9 = 2$ (b) $\log_{10} 10{,}000 = 4$ (c) $\log_{10} 0.01 = -2$ (d) $\log_e b = a$

Exercise 2

(a) 2 (b) 1/2 (c) 0 (d) 5 (e) 1

Exercise 3

1.(a) $\log_4 12$ (b) $\log_{10} \frac{45}{4}$ (c) 3 (d) 0 (e) $\log_{10} b$

2. (a) $x = 2$ (b) $x = 27$