RMIT UNIVERSITY

ILS 1.2 FRACTIONAL INDICES

Previously we considered integer indices. What does a fractional index mean?

The index laws apply to fractional indices as well as positive and negative integer indices.

Using the first index law we know that

$$3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^1$$

That is $3^{\frac{1}{2}}$ multiplied by itself equals 3.

The square root of 3 is the number that, when multiplied by itself, equals 3 and is written as $\sqrt{3}$.

$$\sqrt{3} \times \sqrt{3} = 3^1$$

Since $3^{\frac{1}{2}}$ behaves like $\sqrt{3}$ we say that $3^{\frac{1}{2}} = \sqrt{3}$.

Similarly: $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$ and $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 2$ using the first index law

Since $2^{\frac{1}{3}}$ behaves like the cube root $\sqrt[3]{2}$; we say $2^{\frac{1}{3}} = \sqrt[3]{2}$.

In general: $a^{\frac{1}{n}} = \sqrt[n]{a} \qquad \left(n^{th} \text{ root of } a\right)$ where *n* is a positive integer.

Examples

(1). $4^{\frac{1}{2}} = \sqrt{4} = 2$ (2). $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ (3). $3^{\frac{1}{4}} = \sqrt[4]{3}$

(4). $b^{\frac{1}{5}} = \sqrt[5]{b}$ (5). $x^{\frac{1}{2}} = \sqrt{x}$ (6). $32^{-\frac{1}{5}} = \frac{1}{32^{\frac{1}{5}}} = \frac{1}{\sqrt[5]{32}} = \frac{1}{2}$

In most cases the root of a number will not be able to be written as a fraction and will be an irrational number. For example $\sqrt{2} = 1.414$..........

See Exercise 1

Expressions of the form $a^{\frac{m}{n}}$

If $a^{\frac{1}{n}} = \sqrt[n]{a}$ (n^{th} root of a), what does $a^{\frac{2}{3}}$ mean?

$a^{\frac{2}{3}}$ can be written $\left(a^2\right)^{\frac{1}{3}}$ .....using the third index law

So $a^{\frac{2}{3}} = a^{\frac{2}{3}} = \sqrt[3]{a^2}$ ......using the definition of indices as roots

In general $$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$ where m and n are integers.

Examples

(1). $5^{\frac{3}{4}} = \sqrt[4]{5^3}$ (2). $7^{\frac{5}{2}} = \sqrt{7^5}$ (3). $a^{\frac{7}{5}} = \sqrt[5]{a^7}$

(4). $y^{-\frac{3}{4}} = \frac{1}{y^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{y^3}}$ (5). $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

The index laws can be used to evaluate and simplify expressions with fractional indices.

Examples

Simplify the following, and evaluate if possible.

(1) $\left(x^3y^9\right)^{\frac{2}{3}}$; $\left(x^3y^9\right)^{\frac{2}{3}} = x^{3\times\frac{2}{3}}y^{9\times\frac{2}{3}} = x^2y^6$ ...... using third index law

(2) $3^{\frac{1}{3}} \div 3^{\frac{4}{3}}$; $3^{\frac{1}{3}} \div 3^{\frac{4}{3}} = 3^{\frac{1}{3}-\frac{4}{3}} = 3^{-1} = \frac{1}{3}$ .... using second index law

(3) $32^{\frac{3}{5}}$; $32^{\frac{3}{5}} = \left(2^5\right)^{\frac{3}{5}} = 2^3 = 8$ ....write 32 as 2^5. Use third index law.

(4) $25^{-\frac{1}{2}}$; $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ ....power $\frac{1}{2}$ - same as the square root.

(5) $\left(a^2b^5\right)^{\frac{1}{3}} \times a^{\frac{1}{3}}b^{-\frac{2}{3}}$

$$\left(a^2b^5\right)^{\frac{1}{3}} \times a^{\frac{1}{3}}b^{-\frac{2}{3}} = a^{\frac{2}{3}}b^{\frac{5}{3}} \times a^{\frac{1}{3}}b^{-\frac{2}{3}}$$
$$= a^{\frac{2}{3}+\frac{1}{3}}b^{\frac{5}{3}-\frac{2}{3}}$$
$$= ab$$

See Exercise 2

Exercise 1

Evaluate the following expressions. If the answer is not exact give the decimal approximation to two decimal places.

(a) $64^{\frac{1}{2}}$ (b) $125^{\frac{1}{3}}$ (c) $36^{\frac{1}{4}}$ (d) $81^{-\frac{1}{2}}$

(b) (e) $128^{-\frac{1}{7}}$ (f) $250^{\frac{1}{5}}$

Exercise 2

Simplify the following expressions. Give your answer in index notation with positive indices.

(a) $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ (b) $\sqrt{5}\times\sqrt[3]{5}\times\sqrt[6]{5}$ (c) $\sqrt{a}\times\sqrt[4]{a}\times\sqrt[3]{a^2}$

(d) $\left(125a^6b\right)^{-\frac{1}{3}}\times b^{\frac{2}{3}}$ (e) $\frac{\left(2xy^3\right)^{\frac{1}{2}}}{2}\times\left(\frac{x^{\frac{3}{2}}}{y^2}\right)^4$ (f) $2^{\frac{5}{2}}-2^{\frac{3}{2}}$

Answers

Exercise 1

(a) 8 (b) 5 (c) 2.45 (d) $\frac{1}{9}$ (e) $\frac{1}{2}$ (f) 3.02

Exercise 2

(a) $\frac{2^2}{3^2}$ (b)5 (c) $a^{\frac{17}{12}}$ (d) $\frac{b^{\frac{1}{3}}}{5a^2}$ (e) $\frac{x^{\frac{13}{2}}}{2^{\frac{1}{2}}y^{\frac{13}{2}}}$ (f) $2^{\frac{3}{2}}$