FA1.1: FACTORISATION: COMMON FACTORS

Expansion of brackets (or removing brackets) in an algebraic expression is done by multiplying all the terms inside the brackets by the term(s) outside the brackets.

\[ 5a(3y - 2x) = 15ay - 10ax \]
Each term inside the brackets has been multiplied by 5a.

Factorisation is the reverse of expansion. To factorise a number or algebraic expression means to write the number or expression as a product (multiplication).

Examples
1. \(-2xyz\) has factors \(-2, x, y,\) and \(z\)
2. \(7(a + b)\) has factors 7 and \((a + b)\)
3. \(3(x - y)\) has factors 3 and \((x - y)\)
4. \(x(3a + 2b)\) has factors \(x\) and \((3a + 2b)\)
5. \(2p(2m + 1)(s + 4)\) has factors 2, \(p\), \((2m + 1)\) and \((s + 4)\)

**Expansion means removing brackets**

**Factorisation means inserting brackets**

Factorisation by removing a common factor

- Search each term in the expression for a common factor (every term must have this factor)
- There may be several common factors. Search until you have found all of them
- If there is more than one common factor multiply them to give highest common factor (HCF)
- The HCF is placed before the bracket
- The terms inside the bracket are found by dividing each term by the HCF.
Examples

1. \(5y + 10\)
   \[5y + 10 = 5 \times y + 5 \times 2\]
   common factor of 5

2. \(3x + 3y\)
   \[3x + 3y = 3 \times x + 3 \times y\]
   \[= 3(x + y)\]
   common factor 3

3. \(p^2 + p\)
   \[p^2 + p = p \times p + p \times 1\]
   \[= p(p + 1)\]
   common factor \(p\)

4. \(7y^2 + 7y\)
   \[7y^2 + 7y = 7y \times y + 7y \times 1\]
   \[= 7y(y + 1)\]
   common factors 7 and \(y\) HCF = 7y

5. \(2ab - 12a\)
   \[2ab - 12a = 2a \times b - 2a \times 6\]
   \[= 2a(b - 6)\]
   common factor 2, \(a\), and \(c\). HCF = 2ac

See Exercise 1

A common factor may be negative.

Examples

\[-2a - 2b\]
\[-2a - 2b = (-2) \times a + (-2) \times b\]
\[= -2(a + b)\]
common factor - 2

\[-3x + 6xy\]
\[-3x + 6xy = (-3x) \times 1 - (-3x) \times 2y\]
\[= -3x(1 - 2y)\]
HCF = - 3x

See Exercise 2
Exercises

Exercise 1.

Factorise the following (if possible)

a) $3x + 3y$  
   b) $2a - 2b$  
   c) $8a - 8b + 8c$

d) $xy - 5x$  
   e) $x^2 - x$  
   f) $7x + 21y$

g) $5m - 2n$  
   h) $c^2 - 2bc - 3c$  
   i) $5mn - 10n$

j) $3m^2 - 3mn$  
   k) $7x + 21x^2$  
   l) $12m^2 - 18mn$

m) $5xy - 10xz$  
   n) $5pq - pq^2 - 3pqr$  
   o) $2abc + 6abc^2$

p) $rst + 5rst^2 - 2rs$  
   q) $5mn + 10m - pqr$  
   r) $5xyz - x^2yz^2 + 10x$

Exercise 2

Factorise the following by removing a negative factor.

a) $-3x - 6y$  
   b) $-15xy + 25xz$  
   c) $-2xy + 4xyz$

d) $14xyz - 7xy$  
   e) $-6xyz - 15yz - 3xy^2z$

Answers

Exercise 1

a) $3(x + y)$  
   b) $2(a - b)$  
   c) $8(a - b + c)$

d) $x(y - 5)$  
   e) $x(x - 1)$  
   f) $7(x + 3y)$

g) no factors  
   h) $c(c - 2b - 3)$  
   i) $5n(m - 2)$

j) $3m(m - np)$  
   k) $7x(1 + 3x)$  
   l) $6m(2m - 3n)$

m) $5x(y - 2z)$  
   n) $pq(5 - q - 3r)$  
   o) $2abc(b + 3c)$

p) $rs(t + 5t^2 - 2)$  
   q) no factors  
   r) $x(5yz - xyz^2 + 10)$

Exercise 2

a) $-3(x + 2y)$  
   b) $-5x(3x - 5z)$  
   c) $-2xy(1 - 2z)$

d) $-7xy(-2z + 1)$  
   e) $-3y(2xz + 5z + xyz)$