FG8 QUADRATIC GRAPHS

A quadratic function has the form $f(x) = ax^2 + bx + c \quad a \neq 0$.
The graph of a quadratic function is called a parabola.
To sketch a parabola find and label:

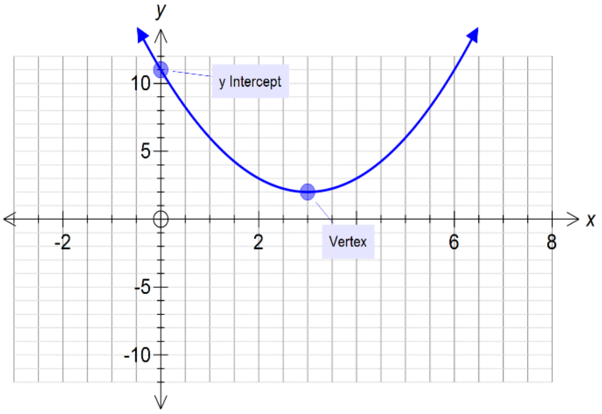
- y-intercept (put $x = 0$)
- x-intercepts (if any) (put $y = 0$, then solve the quadratic equation)
- vertex (turning point)

The coordinates of the vertex are given by:

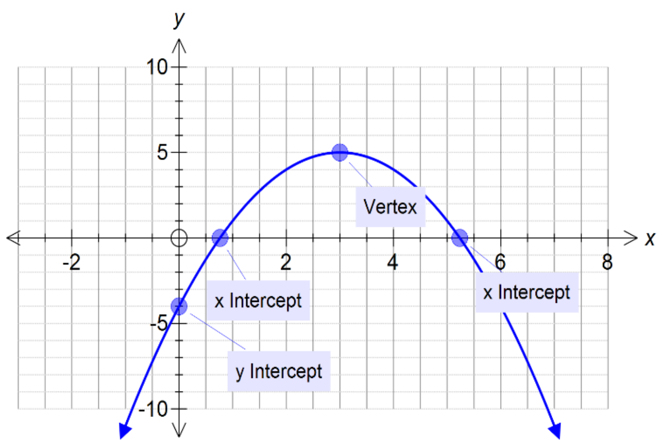
x- co-ordinate: $\left(-\dfrac{b}{2a}\right)$

y – co-ordinate: substitute the value of the x- co-ordinate in the equation for y.

A parabola is symmetrical about a vertical line through the vertex.



If $a > 0$, then the parabola opens upwards.



If $a < 0$, then the parabola opens downwards.

A quadratic function may also be written in turning point form: $\boxed{y = a(x-h)^2 + k}$ where the (h,k) is the turning point.

Examples

$y = (x-3)^2 + 4$ has a turning point at (3,4)

$y = (x+5)^2 + 2$ has a turning point at (–5,2)

$y = 2(x+1)^2$ has a turning point at (–1,0) …… $y = 2(x+1)^2$ can be written as $y = 2(x+1)^2 + 0$

$y = x^2 - 7$ has a turning point at (0, –7) ……. $y = x^2 - 7$ can be written as $y = (x-0)^2 - 7$

$y = 6 - (x-2)^2$ has a turning point at (2,6)…… $y = 6 - (x-2)^2$ can be written as $y = -(x-2)^2 + 6$

See Exercise 1

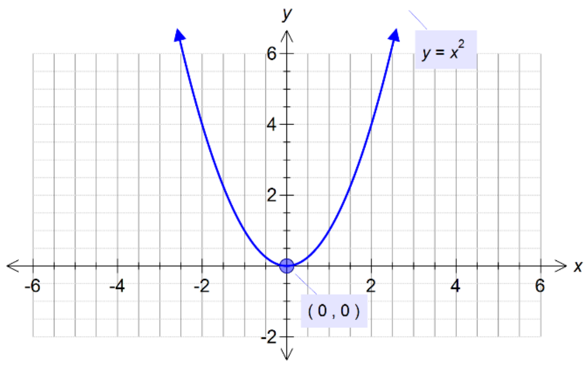
Sketching a parabola

To sketch a parabola find and label:

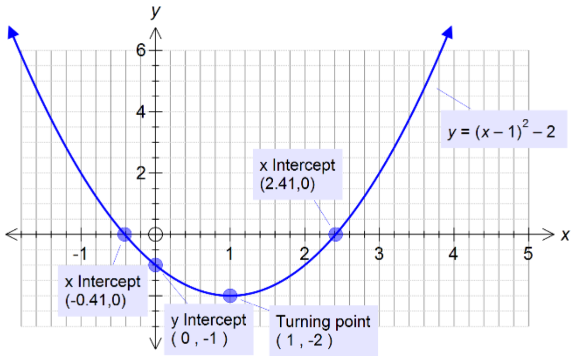
- y-intercept (put x = 0.)
- x-intercepts (if any) (put y = 0, then solve the quadratic equation)
- vertex (turning point)

Examples

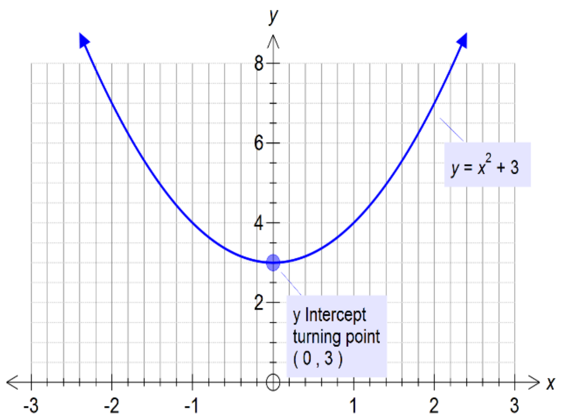
1. Sketch $y = x^2$
   Intercepts x = 0, y = 0
   Turning point (T.P) =(0,0) ... equation in turning point form



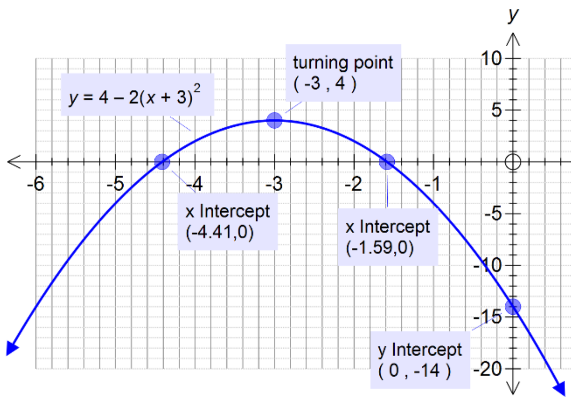
2. Sketch $y = (x-1)^2 - 2$
   y-intercept: $x = 0, y = -1$
   x-intercepts: y = 0, $x = \mp\sqrt{2}+1$
   T.P = (1,–2)) ... equation in turning point form



3. Sketch $y = x^2 + 3$
   y-intercept: $x = 0, y = 3$
   x-intercepts: $y = 0, x^2 + 3 = 0$
   no solution , no x-intercepts
   T.P =(0,3) ... equation in turning point form



4. Sketch $y = 4 - 2(x+3)^2$
   y-intercept: $x = 0, y = -14$
   x-intercepts: y = 0, $(x+3)^2 = 2$
   $x = -3 \pm \sqrt{2}$
   T.P = (– 3,4) ... equation in turning point form



See Exercise 2

5. Sketch $y = x^2 + 2x - 8$

y-intercept: $x = 0, y = -8$
x-intercepts: $y = 0 \quad \therefore\ x^2 + 2x - 8 = 0$
factorise: $(x-2)(x+4) = 0$
x-intercepts $x = 2$ or $x = -4$

T.P: This equation is not in turning point form. Use formula for the x- co-ordinate of turning point: $x = \left(-\dfrac{b}{2a}\right)$

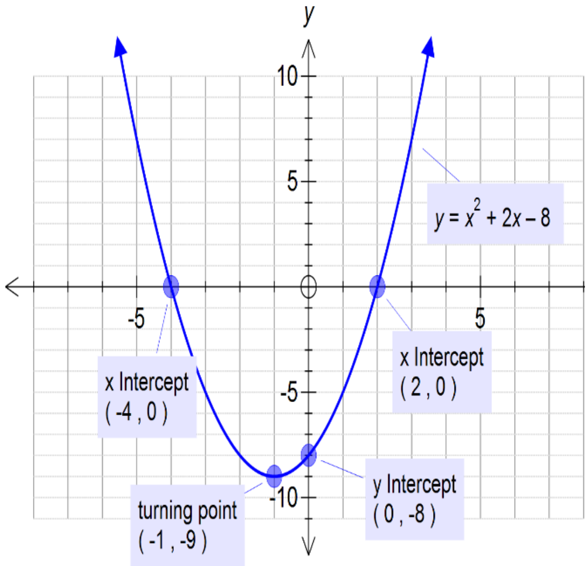
In this equation a = 1, b = 2

$x - \text{coord.} = -\left(\dfrac{2}{2\times1}\right) = -1$

y – coord Substitute x = – 1 in the equation for y.

$y - \text{coord.} = (-1)^2 + 2(-1) - 8 = -9$

$\text{T.P.} = (-1, -9)$



See Exercise 3

Exercise 1

State the turning point of the following graphs.

(a) $y = (x-1)^2 + 5$ (b) $y = 5(x-4)^2 - 12$ (c) $y = (x+2)^2 + 3$

(d) $y = -3(x+5)^2 - 3$ (e) $y = (x-6)^2$ (f) $y = -4x^2 + 3$

Exercise 2

Sketch the graphs of the following:

(a) $y = x^2 - 7$ (b) $y = (x-2)^2 + 1$ (c) $y = 4 - (x+3)^2$

(d) $y = (x-2)^2$ (e) $y = -(x-1)^2 - 1$

Exercise 3

(a) $y = x^2 - x - 6$ (b) $y = -x^2 - 2x + 8$ (c) $y = x^2 - 4x$

(d) $y = -2x^2 - 6x$ (e) $y = x^2 - 9$

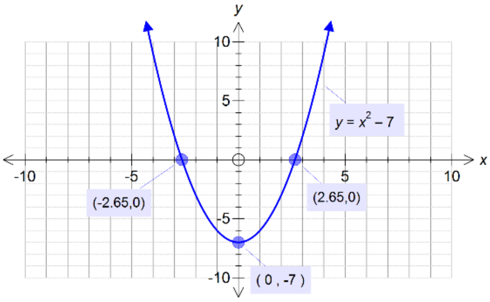
Answers

Exercise 1

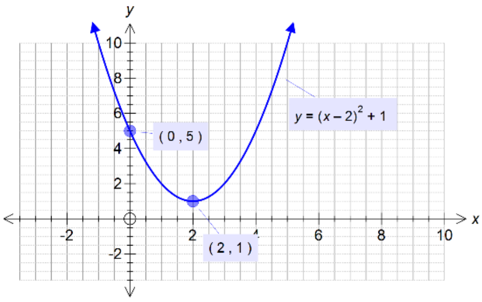
(a) (1, 5)   (b) (4, −12)   (c) (− 2, 3)   (d) (−5, − 3)   (e) (6,0)   (f) (0,3)

Exercise 2

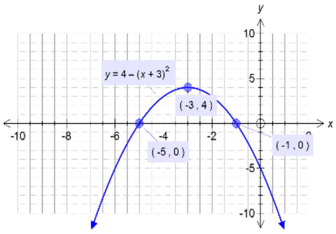
(a)



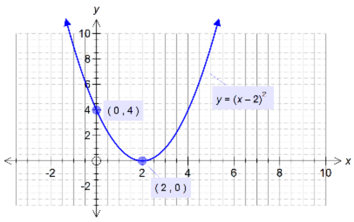
(b)



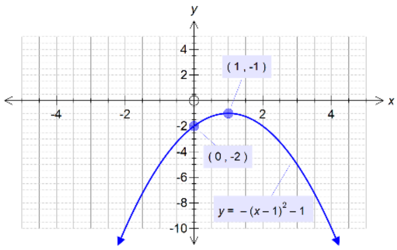
(c)



(d)

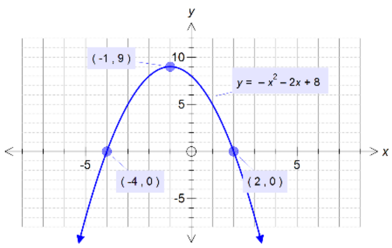


(e)

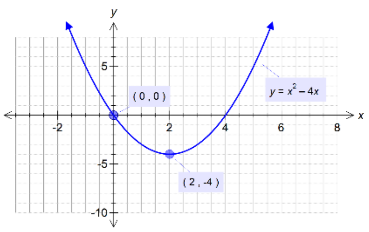


Exercise 3

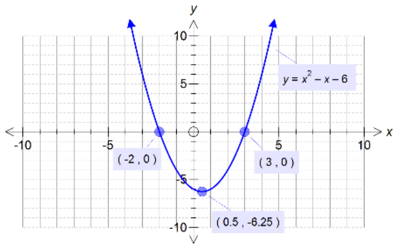
(a)



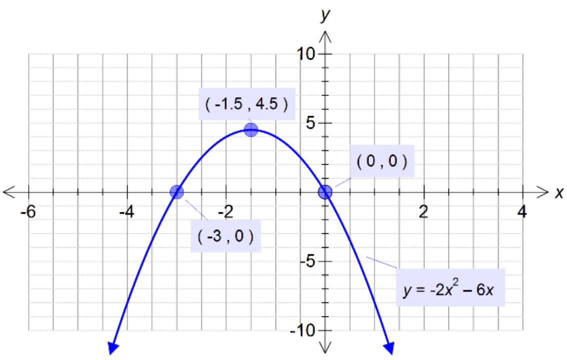
(b)



(c)



(d)



(e)