

STUDY TIPS

FG4 ABSOLUTE VALUE

The absolute value of a number $|x|$ gives a measure of its size or magnitude regardless of whether it is positive or negative. If a number is plotted on a number line then its absolute value can be considered to be the *distance* from zero.

Examples

Eg (i) $|2| = 2$

(ii) $|-2| = 2$

(iii) $|-4 + 3| = |-1| = 1$

(iv) $|-8| + |-1| = 8 + 1 = 9$

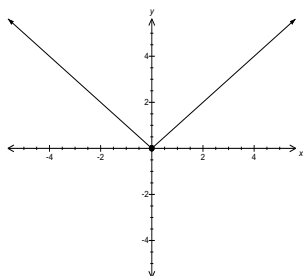
(v) $|x| = 7 \Rightarrow x = 7$ or $x = -7$

The absolute value function and its graph

The absolute value function is a hybrid function defined as follows:

$$f:R \rightarrow R \text{ where } f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

with graph



NB: The domain of $f(x) = |x|$ is R The range is $R^+ \cup 0$

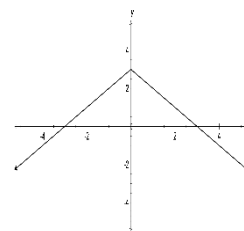
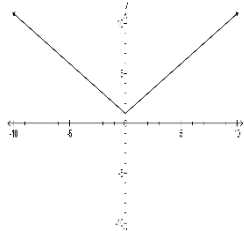
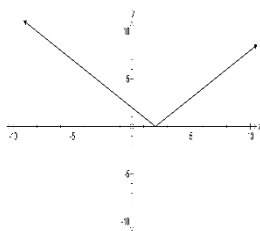
The graph of $y = |x|$ may be translated in the same way as the graphs of other functions.

Compare the graphs of the following functions with that of $y = |x|$

1. $y = |x-2|$

2. $y = |x| + 1$

3. $y = 3 - |x| = -|x| + 3$



$y = |x|$ translated horizontally
2 units to the right

$y = |x|$ translated
vertically one unit up

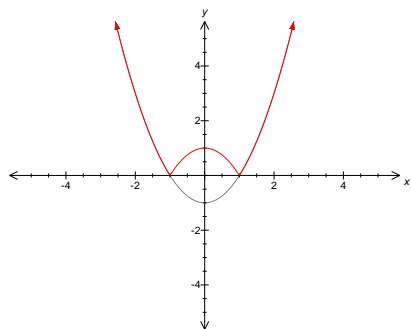
$y = |x|$ reflected in the x-axis
followed by a vertical shift of 3 units

To sketch the graph of $y = |f(x)|$ we need to sketch the graph of $y = f(x)$ first and then reflect in the x-axis the portion of the graph which is below the x-axis.

Example

Sketch $\{(x,y): y = |x^2 - 1|\}$

The graph of this function is the graph of $y = x^2 - 1$ with the portion below the x-axis reflected across the x-axis.



Equations & inequalities involving $|f(x)|$

Because $y = |f(x)|$ is a hybrid function two cases must be considered when solving equations and inequalities

Examples

1. Solve $|x - 2| = 3$

If $|x - 2| = 3$

then $x - 2 = 3$ or $-(x - 2) = 3$

ie $x = 5$ or $x = -1$

With an absolute value expression on each side of the equation it is easier to square both sides:

2. Solve $|2x + 1| = |x - 5|$

If $|2x + 1| = |x - 5|$

then $(2x+1)^2 = (x - 5)^2$

ie $4x^2 + 4x + 1 = x^2 - 10x + 25$

ie $3x^2 + 14x - 24 = 0$

ie $(3x - 4)(x + 6) = 0$

ie $x = \frac{4}{3}$ or $x = -6$

NB: Care must be taken when multiplying or dividing by a negative to reverse an inequality

2. Solve $\left| \frac{2-x}{3} \right| < 4$

$$\left| \frac{2-x}{3} \right| < 4 \Rightarrow |2-x| < 12$$

$$\Rightarrow -12 < 2-x < 12$$

$$\Rightarrow -14 < -x < 10$$

$$\Rightarrow 14 > x > -10 \text{ or } -10 < x < 14$$

Exercise 1

Evaluate

1. $|-11|$

2. $|-9+4|$

3. $-|4|-|-5|$

4. $|-12|-|3|$

5. $|-30| \div |5|$

Exercise 2

Sketch the graph of

1. $y = |x+4|$

2. $y = |x-1|-3$

3. $y = |3-x^2|$

Exercise 3

Find for $x \in \mathbf{R}$

1. $\{x: |x| = 6\}$

2. $\{x: |x-1| < 3\}$

3. $\{x: \left| \frac{x-3}{2} \right| \geq 1\}$

4. $\left\{ \frac{|x|}{2} = |x+2| \right\}$

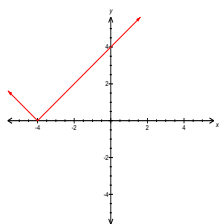
Answers

Exercise 1

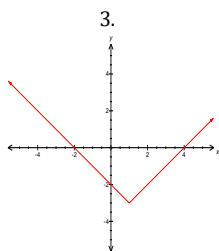
1) 11 2) 5 3) -9 4) 9 5) 6

Exercise 2

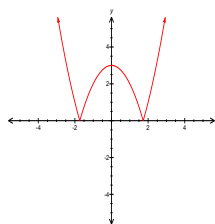
1.



2.



3.



Exercise 3

1. $\{-6, 6\}$

2. $\{x: -2 < x < 4\}$

3. $\{x: x \leq 1\} \cup \{x: x \geq 5\}$

4. $\{-4, -\frac{4}{3}\}$