

FG3 INVERSE FUNCTIONS

Definition of an inverse function

If $f^{-1}(x)$ is the inverse function of a one-to-one function $f(x)$ then $f^{-1}(x)$ is the set of ordered pairs obtained by interchanging the first and second elements in each ordered pair.

So if $(a,b) \in f$ then $(b,a) \in f^{-1}$ and if $f(a) = b$ then $f^{-1}(b) = a$

The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1} .

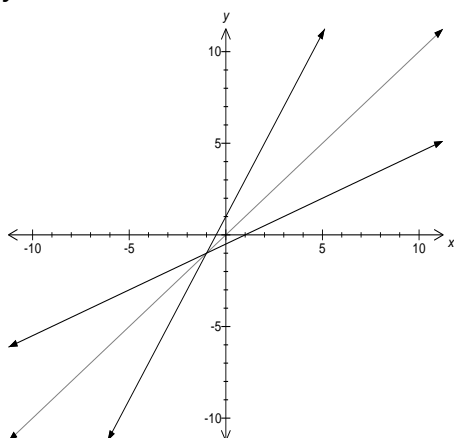
For example the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $y = f(x) = \frac{x-1}{2}$ has an inverse function with rule

$$f^{-1}(x) = 2x + 1.$$

So $(3,1)$ belongs to f and $(1,3)$ belongs to f^{-1} , and $(-7,-4)$ belongs to f and $(-4,-7)$ belongs to f^{-1} .

Graph of an inverse function

The graphs of any one-to-one function f and its inverse f^{-1} are symmetric about the line $y = x$.



Finding an inverse function for $y = f(x)$

To obtain the rule for an inverse function swap the x and y coordinates in f and rearrange to express y in terms of x :

Example

Find the inverse function of f where $f(x) = 2 - 3x$

$$y = 2 - 3x$$

$$x = 2 - 3y \quad [\text{swap } x \text{ and } y]$$

$$x - 2 = -3y \quad [\text{rearrange to make 'y' the subject}]$$

$$-x + 2 = 3y$$

$$\frac{-x + 2}{3} = y$$

$$\therefore f^{-1}(x) = \frac{-x + 2}{3}$$

Exercise

Find the inverse of each of the following one-to-one functions:

1) $y = x + 5$

2) $y = 4x$

3) $y = \frac{2x+1}{3}$

4) $y = \sqrt{2x-1}, x \geq \frac{1}{2}$

Answers

1) $f^{-1}(x) = x - 5$

2) $f^{-1}(x) = \frac{x}{4}$

3) $f^{-1}(x) = \frac{3x-1}{2}$

4) $f^{-1}(x) = \frac{x^2+1}{2}, x \geq \frac{1}{2}$