

FG2 INTERVAL NOTATION

Interval

Often the domain of a function will be restricted to a subset of R .

This subset is called an *interval*, and the *end points* are a and b .

An interval may be represented on a real number line as follows :



In inequality notation the above number line would be written as $a \leq x \leq b$.

In *interval notation* the above interval would be written as $[a, b]$.

Closed Interval

Because the endpoints are included in the interval, this is called a *closed interval*.

Square brackets are used. eg. $[2, 5]$.

The end points on the on the real number line are represented as solid circles (or square brackets).

Open interval

If the endpoints are excluded, the interval is an *open interval*.

Curved brackets are used. eg $(2, 5)$.









The end points on the real number line are represented as open circles (or curved brackets).



This is written in inequality notation as $a < x < b$.

In interval notation as (a, b) .

Examples

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \leq x \leq b$	
(a, b)	$a < x < b$	
$[a, b)$	$a \leq x < b$	
$(a, b]$	$a < x \leq b$	
$[a, \infty)$	$x \geq a$	
(a, ∞)	$x > a$	
$(-\infty, b]$	$x \leq b$	
$(-\infty, b)$	$x < b$	

In interval notation the smaller number is **always** written to the left. i.e. $[-3, 5] \neq (5, -3]$

Note: the symbol ∞ (infinity) is **not** a numeral.

∞ is the concept of continuing indefinitely to the right; $-\infty$ is the concept of continuing indefinitely to the left.

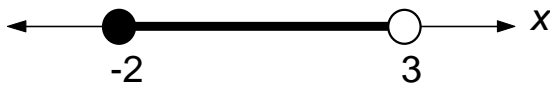
Hence we cannot write $[b, \infty]$, $[-\infty, a]$ or $b \leq x \leq \infty$, $-\infty \leq x \leq a$ etc..

Examples

1. Write the following in inequality notation and graph on a real number line.

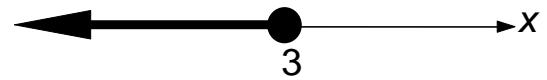
(a) $[-2, 3)$ Inequality notation $-2 \leq x < 3$

Graph



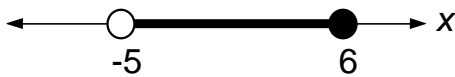
(b) $(-\infty, 3]$ Inequality notation $x \leq 3$

Graph



2. Write the interval notation and inequality notation for the following line graphs.

(a)



Interval notation $(-5, 6]$.

Inequality notation $-5 < x \leq 6$

(b)



Interval notation $(10, \infty)$

Inequality notation $10 < x < \infty$

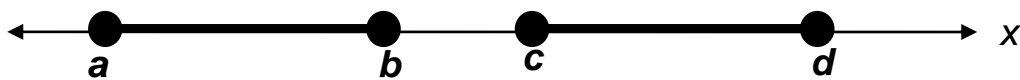
See Exercise 1

Two intervals

Two (or more) subsets of \mathbb{R} , with end points a and b , and c and d , respectively, can also be represented on a real number line.

Examples

1.

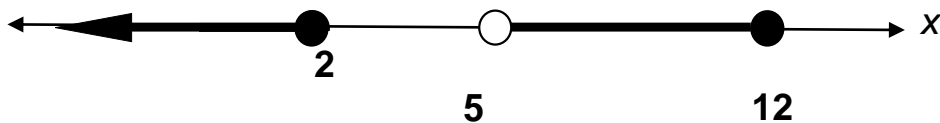


This is written in interval notation as $[a, b] \cup [c, d]$.

The symbol as \cup represents '*in union with*'

In inequality notation this may be written: $a \leq x \leq b$ with $c \leq x \leq d$

2.



This is written in interval notation as $(-\infty, 2] \cup (5, 12]$.

In inequality notation this may be written: $x \leq 2$ with $5 < x \leq 12$

See Exercises 2 and 3

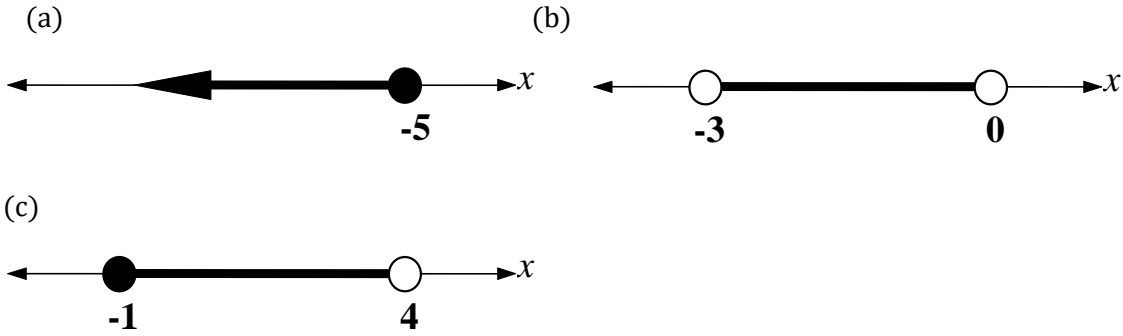
Exercise 1

Write the following inequalities in interval notation and graph on a real number line.

- (a) $1 \leq x \leq 10$ (b) $-6 \leq x < -4$ (c) $x > 5$

Exercise 2

Write the following in interval notation and inequality notation.



Exercise 3

Graph the following on a real number line and write in inequality notation.

- (a) $(-\infty, -3) \cup (8, 13]$. (b) $[-1, 4] \cup [6, 9]$. (c) $(-\infty, 3] \cup (6, \infty)$

Answers

Exercise 1

- (a) $[1, 10]$



- (b) $[-6, -4)$



- (c) $(5, \infty)$

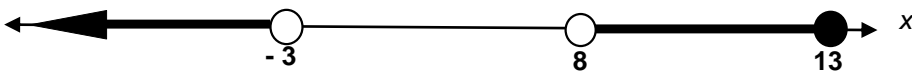


Exercise 2

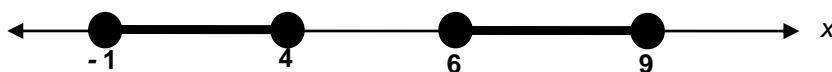
- (a) $(-\infty, -5], x \leq -5$ (b) $(-3, 0), -3 < x < 0$ (c) $[-1, 4), -1 \leq x < 4$

Exercise 3

- (a) $-\infty < x < -3$ with $8 < x \leq 13$



- (b) $-1 \leq x \leq 4$ with $6 \leq x \leq 9$



- (c) $-\infty < x \leq 3$ with $6 < x < \infty$

