# FG1: FUNCTIONS AND RELATIONS

**Relations**

A relation is a set of *ordered pairs*.

For example (1, 2), (2, 6), (3, 4), $(x, y)$ are ordered pairs.

The ***domain*** of a relation is the set of first elements or the ***x-values*** of the ordered pairs.

For the above ordered pairs the domain, $\mathbf{d_m} = \{1, 2, 3, x\}$.

The ***range*** of a relation is the set of second elements or the ***y-values*** of the ordered pairs.

For the above ordered pairs the range, $\mathbf{r_g} = \{2, 6, 4, y\}$.

There is often a *rule* that links the domain and range.

$$S = \{(x, y):\ y > x,\ x \in R\}$$

The set of — ordered pairs — the rule joining the domain and range. — any restrictions placed on the domain or range.

This relation, called *S*, says that all ordered pairs that have *y* greater than *x* are included in the relation.

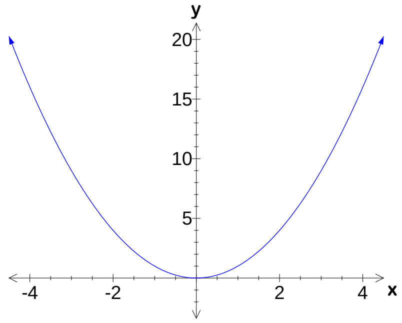
| | |
|---|---|
| The domain of this relation is, $d_m = R$ | $x \in R$ means $x$ belongs to R |
| The range of this relation is, $r_g = R$. | $R$ is the set of real numbers |

**Example 1**

Sketch the graph of the following relation and state the domain and range.

$$\{(x, y):\ y = x^2\}$$

In this example the rule joining the set of ordered pairs $(x,y)$ is $y = x^2$




$x$ can be any real number
Domain = R

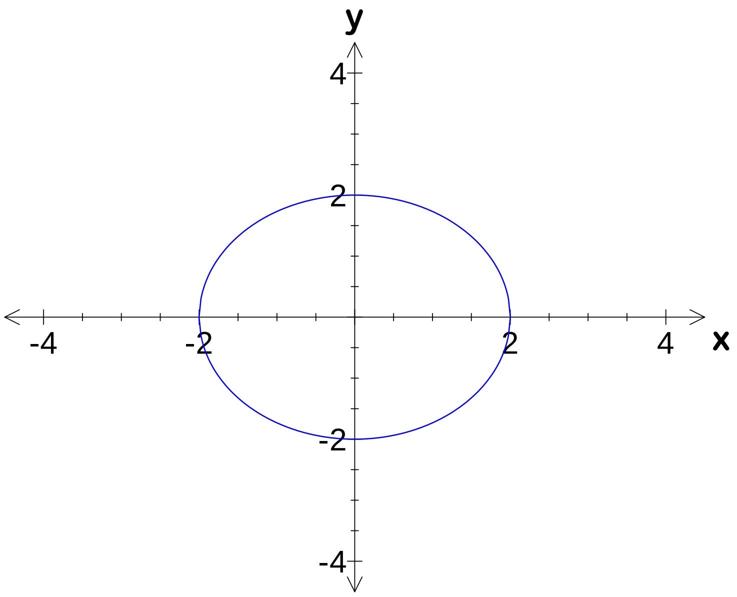
$y$ must be $\geq 0$
Range is $\{y : y \geq 0\}$

**Example 2**

Sketch the graph of $x^2 + y^2 = 4$

State the domain and range of this relation.

In this example the rule joining the set of ordered pairs $(x,y)$ is $x^2 + y^2 = 4$




From the graph it can be seen that:

The domain is $\{x : -2 \le x \le 2\}$ The range is $\{y : -2 \le y \le 2\}$

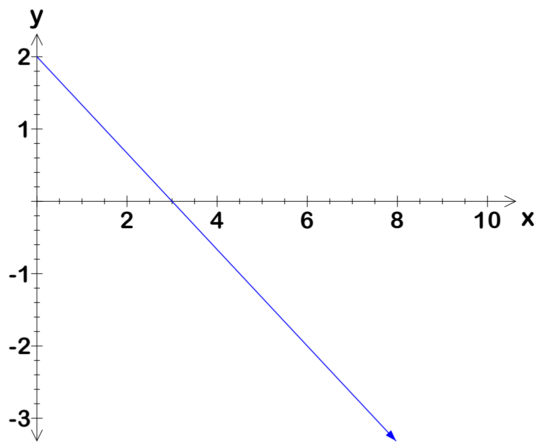
**Example 3**

Sketch the graph of $\{(x,\ y): 2x + 3y = 6,\ x \ge 0\}$

State the domain and range of this relation.

In this example the rule joining the set of ordered pairs $(x,y)$ is $2x + 3y = 6$

The restriction $x \ge 0$ is placed on the domain.

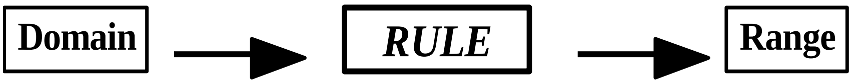



The restriction $x \ge 0$ is specified in the statement of the relation.

If $x = 0,\ y = 2$ ( from the relation $2x + 3y = 6$ ).

Therefore, The domain is $\{x : x \ge 0\}$ The range is $\{y : y \le 2\}$

The rule of a relation may be thought of as:




Values taken from the domain ***produce*** values for the range, after passing through the rule that defines the relation.

***See Exercise 1***

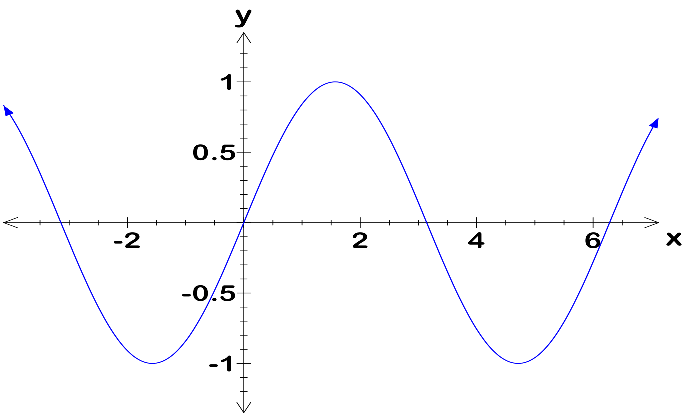
**Functions**

From some of the previous examples it can be seen that some values in the domain (*x*-values) may have many, or even an infinite number of corresponding values in the range (*y*-values).

A ***function*** is a special type of relation.
Each point in the domain of a function has a unique value in the range.
Every value of $x$ may have only one value of $y$.

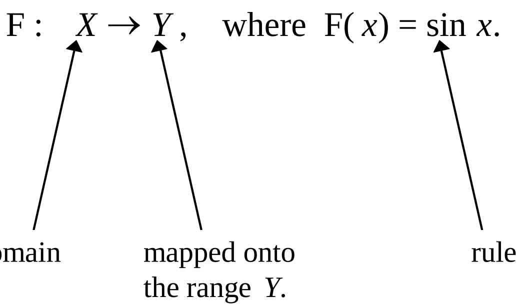
**Example 1**

$F = \{(x, y) : y = \sin x, x \in R\}$




If we choose any possible value of $x$, there exists only one corresponding value of $y$.
$\therefore$ The relation $F$ is a function.

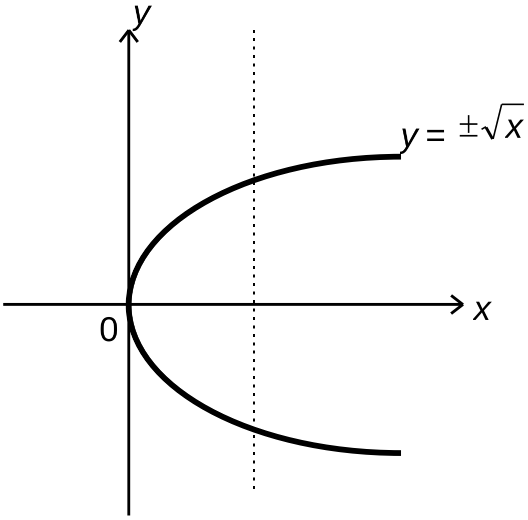
Another way of writing this function is with *mapping notation.*



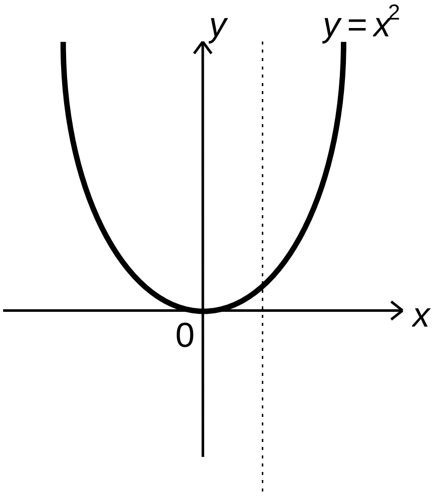

If only the rule is given then we assume that the domain is $R$.

When relations are represented graphically a *vertical line* test may be applied to decide if they are functions

1.




not a function – x value has two corresponding y values.




a function –x value has only one corresponding y value.

2. The relation $(-1,2),(-1,4),(1,6),(2,8),(3,10)$ is **not a function** because the value $x=-1$ has two corresponding values for *y*. (2 and 4)

3. The relation $(-1,1),(0,2),(1,3),(2,5),(3,7)$ **is a function** because each *x* value has only one corresponding *y* value.

***See Exercise 2***

## Implied Domain

If only the rule of a function is given then we assume that the domain is *R* unless otherwise defined implicitly by the function.

**Examples**

**1.** If a function involves a square root the domain, in the real number system, is restricted to those values of *x* that result in a positive number or zero under the square root sign.

The domain of the function $y=+\sqrt{x-4}$ is restricted such that $x-4\geq 0$; The domain is $\{x : x \geq 4\}$

The domain of the function $y=+\sqrt{9-x^2}$ is restricted such that $9-x^2\geq 0$; The domain is $\{x : -3 \leq x \leq 3\}$

**2.** If the function involves a fraction the value the in the denominator must not equal zero.

The domain of the function $y=\dfrac{3}{x+5}$ is restricted such that $x+5\neq 0$;

the domain is $\{x : x$ is a real number and $x \neq -5\}$.

The domain of the function $y=\dfrac{3}{2x-8}$ is restricted such that $2x-8\neq 0$;

the domain is $\{x : x$ is a real number and $x\neq 4\}$.

***See Exercise 3***

**Exercise 1.**

State the domain and range of the following relations.

(a) $(-2,1),(0,2),(2,5),(2,7),(3,9)$ (b) $(4,1),(5,2),(6,3)$

(c) $x^2+y^2=25$ (d) $\{(x,\ y):2y=6-5x,\ x\geq 2\}$

**Exercise 2**

Which of the following relations are functions?

(a) $\{(x,\ y):y=2x+4,\ \}$ (b) $\{(x,\ y):y=4-x^2\}$

(c) $\{(x,\ y):x^2+y^2=36\}$ (d) $\{(x,\ y):y=7\}$

(e) $\{(x,\ y):x=-2\}$ (g) $\left\{(x,\ y):y=-\sqrt{4-x^2}\right\}$

**Exercise 3**

State the domain of the following functions.

(a) $\{(x,\ y):y=x+2\}$ (b) $\{(x,\ y):y=4-x^2\}$

(c) $\left\{(x,\ y):y=+\sqrt{4-x}\right\}$ (d) $\left\{(x,\ y):y=\dfrac{3}{x+2}\right\}$

(e) $\left\{(x,\ y):y=\dfrac{5}{\sqrt{x\ -7}}\right\}$ (f) $\left\{(x,\ y):y=\dfrac{1}{x+2}-\dfrac{3}{x-4}\right\}$

**Answers**

**Exercise 1**

(a) dm = {- 2, 0 2, 3}. rg = {1, 2, 5, 7, 9}. (b) dm = {4, 5, 6}. rg = {1, 2, 3}.

(c) dm = $\{x: -5\leq x\leq 5\}$ rg = = $\{y: -5\leq y\leq 5\}$

(d) dm = = $\{x:\ x\geq 2\}$ rg = = $\{y: y\leq -2\}$

**Exercise 2**

(a), (b), (d), (g)

**Exercise 3**

(a) $\{R\}$ (b) $\{R\}$ (c) $\{x\leq 4\}$ (d) $\{x:\ x$ *is a real number and* $x\neq 2\}$

(e) $\{x:\ x>7\}$ (f) $\{x: x$ *is a real number and* $x\neq -2$ *and* $x\neq -4\}$