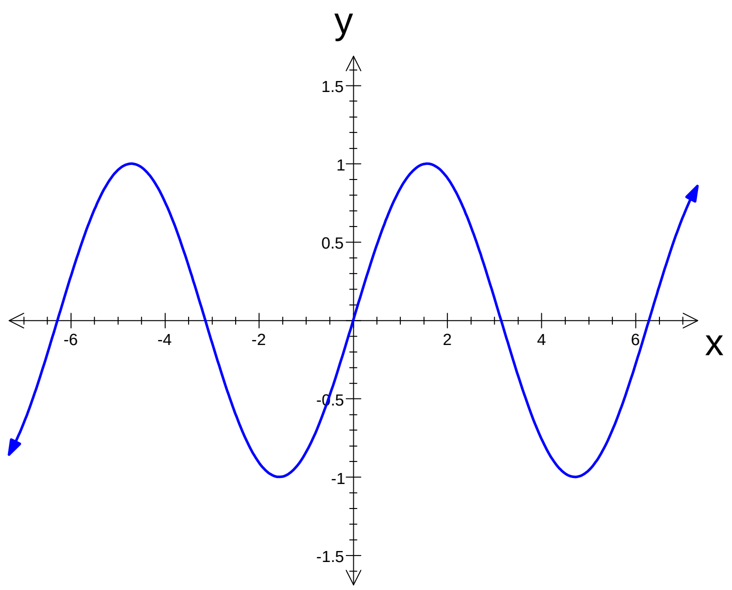
FG10 GRAPHS OF SIN AND COS FUNCTIONS

Both the functions $y = \sin x$ and $y = \cos x$ have a domain of R and a range of $[-1,1]$ The graphs of both functions have an **amplitude of 1** and **a period of** 2π **radians**. (repeats every 2π units).

(Remember $\pi \approx 3.142$ so $2\pi \approx 6.284$)

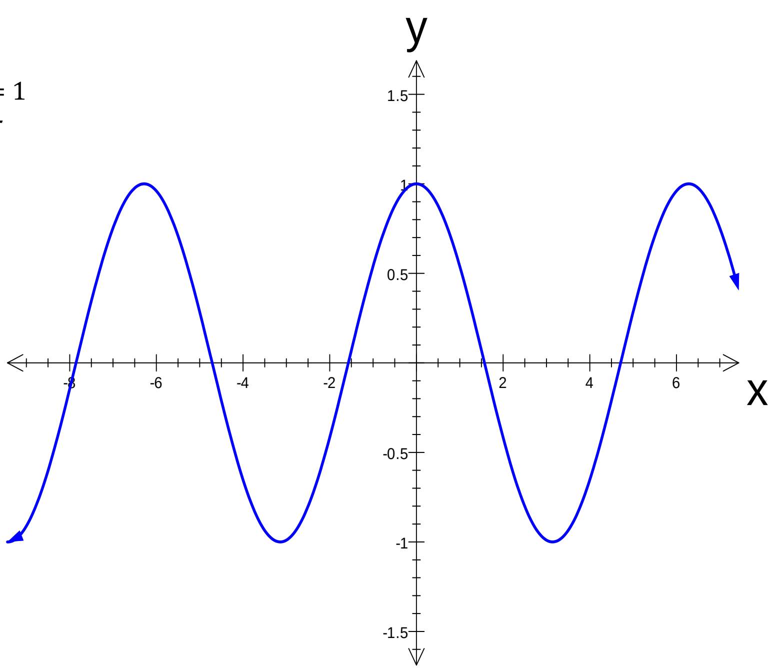
Sine function y = sin(x)

$y = \sin x$
Amplitude $= 1$
Period $= 2\pi$



Cosine function y = cos(x)

$y = \cos x$
Amplitude $= 1$
Period $= 2\pi$



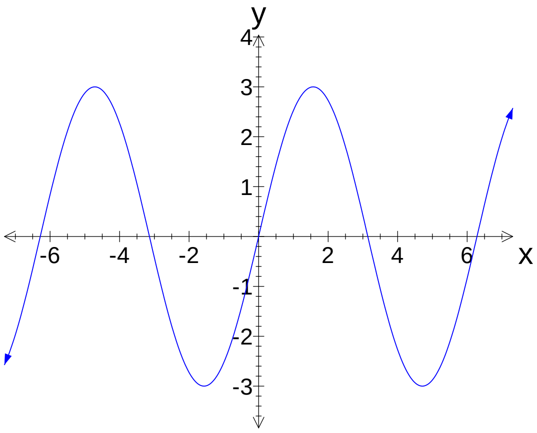
$y = a\sin(nx)$, $y = a\cos(nx)$

The graphs of both $y = a\sin nx$ and $y = a\cos nx$ have:

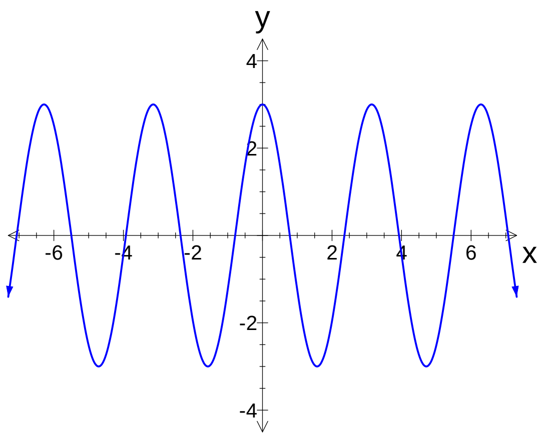
amplitude = 'a'   period = $\dfrac{2\pi}{n}$

Examples

$y = 3\sin x$



$y = 3\cos 2x$



$y = 3\sin x$ has:
an **amplitude of 3** $(a = 3)$
a period of 2π , $(n = 1)$

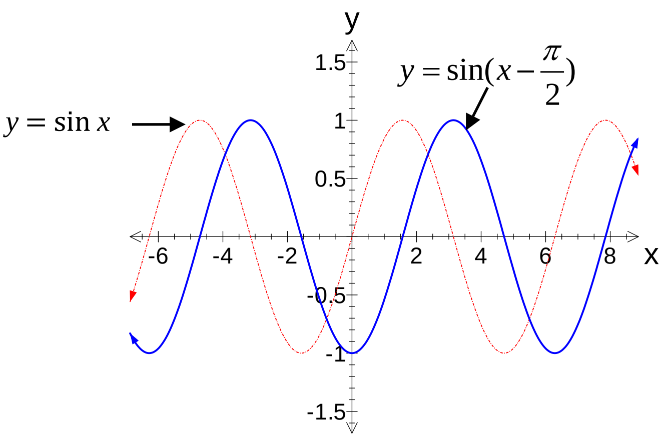
$y = 3\cos 2x$ has:
an **amplitude of 3** $(a = 3)$
a **period of** $\dfrac{2\pi}{2} = \pi$, $(n = 2)$

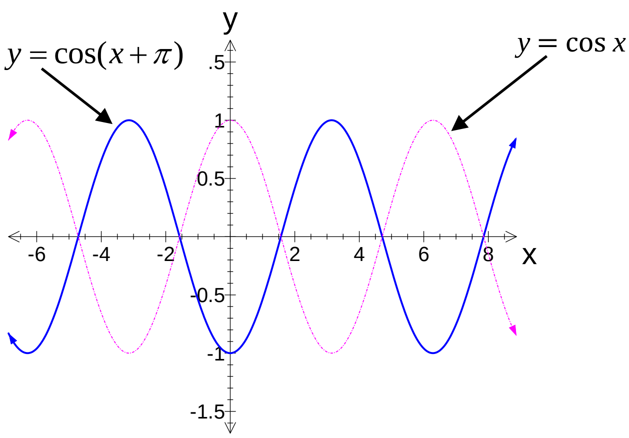
$y = a\sin(x-\phi)$, $y = a\cos(x-\phi)$

Replacing x with $(x-\phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units right.

$y = a\sin(x+\phi)$, $y = a\cos(x+\phi)$

Replacing x with $(x+\phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units left.





$y = \sin(x - \frac{\pi}{2})$ is the graph of $y = \sin x$ (dotted red), shifted $\frac{\pi}{2}$ (≈ 1.571) **right.**

Amplitude = 1 Period = 2π

$y = \cos(x + \pi)$ is the graph of $y = \cos x$ (dotted red), shifted π (≈ 3.142) **left.**

Amplitude = 1 Period = 2π

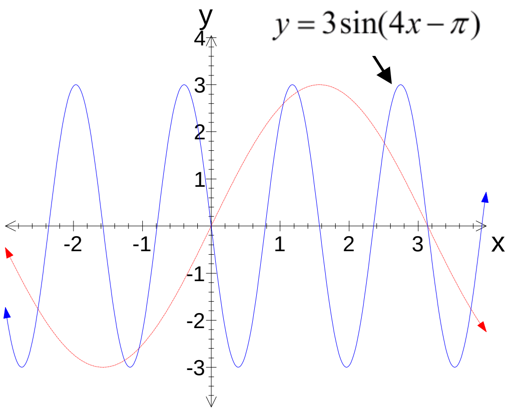
Examples

$y = a\sin(nx - \phi)$, $y = a\cos(nx - \phi)$

To graph $y = a\sin(nx - \phi)$ or $y = a\cos(nx - \phi)$ first rearrange to the form $y = a\sin n(x - \frac{\phi}{n})$ or $y = a\cos n(x - \frac{\phi}{n})$ respectively by taking a factor of 'n' from $(nx - \phi)$.

The graph of $y = a\sin n(x - \frac{\phi}{n})$ has **amplitude 'a', period $\frac{2\pi}{n}$ and shifted horizontally $\frac{\phi}{n}$ units right** from the basic sine graph with amplitude a and period $\frac{2\pi}{n}$	The graph of $y = a\cos n(x - \frac{\phi}{n})$ has **amplitude 'a', period $\frac{2\pi}{n}$ and shifted horizontally $\frac{\phi}{n}$ units right** from the basic cosine graph with amplitude a and period $\frac{2\pi}{n}$

Example



For example to sketch the graph of $y = 3\sin(4x - \pi)$ rearrange to:

$y = 3\sin 4(x - \frac{\pi}{4})$.

The period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The amplitude is 3.

There is a horizontal shift right of $\frac{\pi}{4}$.

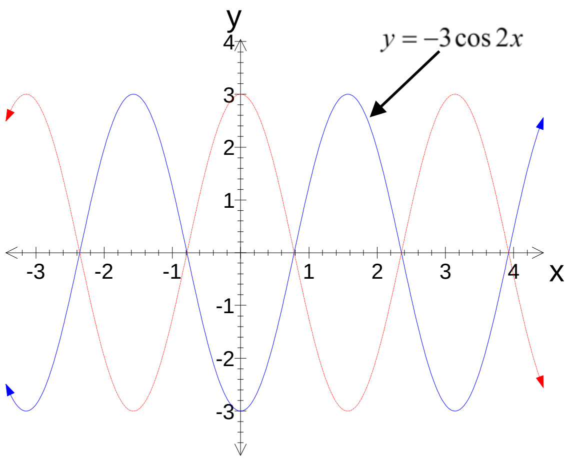
Dotted red graph is $y = 3\sin 4(x)$

Reflection

If the coefficient of the function is negative the graph will be reflected about the x – axis.
The amplitude remains positive.

Example

$y = -3\cos 2x$



The graph of $y = -3\cos 2x$ has an amplitude of 3 and a period of π.

It is a reflection of the graph of $y = 3\cos 2x$, (dotted red), about the x – axis.

Exercises

1.Sketch the graph the following for one complete cycle stating the amplitude and period:

(a) $y = 2\cos x$ (b) $y = 2\sin 3x$ (c) $y = \frac{1}{2}\sin 2x$ (d) $y = 3\cos\frac{x}{2}$ (e) $y = -2\sin 3x$

2. Sketch the graph the following for one complete cycle stating the amplitude and period.

(a) $y = 2\sin(x - \pi)$ (b) $y = \cos(x + \frac{\pi}{2})$

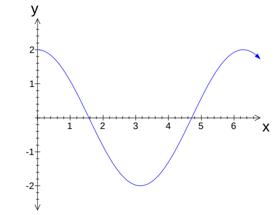
3. Sketch the graph the following for one complete cycle stating the amplitude and period.

(a) $y = 2\sin(3x - \pi)$ (b) $y = 3\cos(4x - 2\pi)$ (c) $y = 2\sin(2x + \frac{\pi}{3})$

Answers

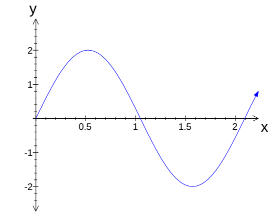
1(a)

Amplitude $= 2$. Period $= 2\pi$



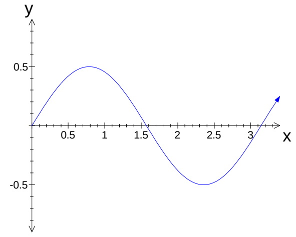
1(b)

Amplitude $= 2$. Period $= 2\pi/3$



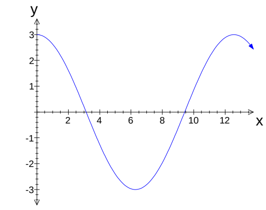
1(c)

Amplitude $= 0.5$. Period $= \pi$



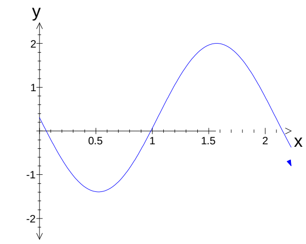
1(d)

Amplitude $= 3$. Period $= 4\pi$



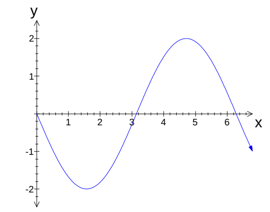
1(e)

Amplitude $= 2$. Period $= 2\pi/3$



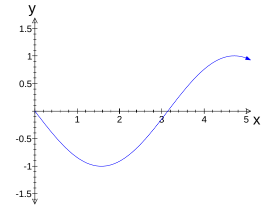
2(a)

Amplitude $= 2$. Period $= 2\pi$



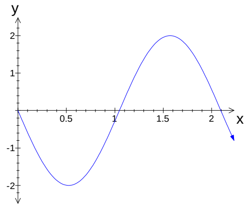
2(b)

Amplitude $= 1$. Period $= 2\pi$



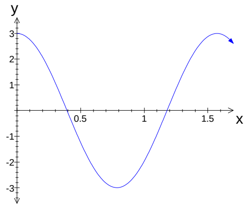
3(a)

Amplitude $= 2$. Period $= 2\pi/3$



3(b)

Amplitude $= 3$. Period $= \pi/2$



3(c)

Amplitude $= 2$. Period $= \pi$