ENST2.6: CENTROIDS

Question 1 Calculate the centroid \((\bar{x}, \bar{y})\) of the shaded area. (Hibbeler, R.C, 2010. Statics, Pearson)

![Centroid of an area diagram]

**Worked Solution 1**

A horizontal strip of area \(dA = x \, dy\) is chosen as the calculations are easier.

\[ \bar{x} = \frac{\int_{A} \bar{x} \, dA}{\int_{A} dA} = \frac{\int_{0}^{1} x \bar{x} \, dA}{\int_{0}^{1} dA} = \int_{0}^{1} (x \bar{x} \, dy) \]

where \(\bar{x} = \frac{x}{2}\)

Since \(y = x^3\), \(x = y^{\frac{1}{3}}\). Substituting into above for \(x\):

\[ \bar{x} = \frac{\int_{0}^{1} \left(\frac{y^{\frac{1}{2}}}{3}\right) y^{\frac{2}{3}} \, dy}{\int_{0}^{1} y^{\frac{2}{3}} \, dy} = \frac{1}{2} \frac{\int_{0}^{1} y^{\frac{2}{3}} \, dy}{\int_{0}^{1} y^{\frac{2}{3}} \, dy} = \frac{3}{10} \left[ y^{\frac{5}{3}} \right]_{0}^{1} = \frac{3}{10} = 0.4 \]
\[ \bar{y} = \frac{\int_A y \, dA}{\int_A dA} = \frac{\int_0^y (2x \, dy)}{\int_0^x (2x \, dy)} \]

\[ \bar{y} = \frac{\int_0^y \frac{y}{2} dy}{\int_0^x \frac{y}{2} dy} = \frac{\frac{3}{4} \left[ \frac{y^2}{3} \right]_0^y}{\frac{3}{4} \left[ \frac{y^2}{2} \right]_0^y} = \frac{\frac{y}{2}}{\frac{y}{2}} = 0.57 \]

The coordinates of the centroid are \((0.4, 0.57)\)

**Question 2**

Locate the centroid of the composite area shown below with respect to the X- and Y-axes.


**Worked Solution 2**

Divide area into 3 rectangular elements \(A_1, A_2\) and \(A_3\).

![Diagram of composite area and centroid calculation](image)

Centroid of a composite area

\[ \bar{x} = \frac{\sum (A_x)}{\sum (A)} \quad \bar{y} = \frac{\sum (A_y)}{\sum (A)} \]

where \(x\) = horizontal distance to centroids \(A_1, A_2, A_3\) from Y-axis

\(y\) = vertical distance to centroids \(A_1, A_2, A_3\) from X-axis

<table>
<thead>
<tr>
<th>Element</th>
<th>Area (A)</th>
<th>Distance</th>
<th>Area Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600</td>
<td>15</td>
<td>9000</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>5</td>
<td>1500</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>30</td>
<td>18 000</td>
</tr>
<tr>
<td>Σ =</td>
<td>1 500</td>
<td>-</td>
<td>28 500</td>
</tr>
</tbody>
</table>

\[ \bar{x} = \frac{\sum (A_x)}{\sum (A)} = \frac{28 500}{1500} = 19\text{mm} \quad \bar{y} = \frac{\sum (A_y)}{\sum (A)} = \frac{40 500}{1500} = 27\text{mm} \]

The coordinates of the centroid are \((19\text{mm}, 27\text{mm})\)