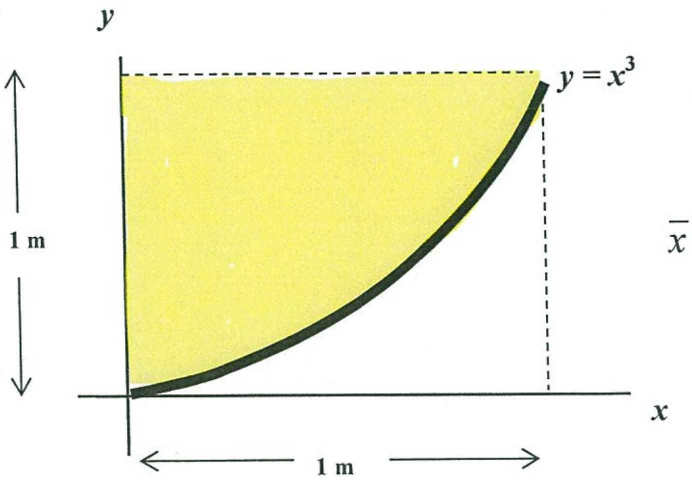
RMIT UNIVERSITY

WORKED SOLUTIONS

ENST2.6: CENTROIDS

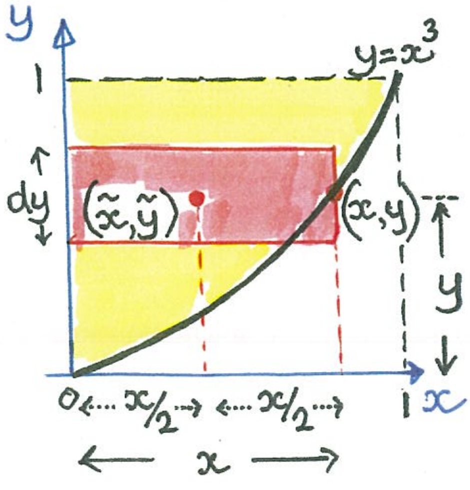
Question 1 Calculate the centroid ($\bar{x}, \bar{y}$) of the shaded area. (Hibbeler, R.C, 2010. *Statics*, Pearson)



Centroid of an area

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

Worked Solution 1



A horizontal strip of area $dA = x\,dy$ is chosen as the calculations are easier.
[A vertical strip would be $dA = \left(\frac{(1-y)}{2}+y\right)dx$]

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^1 (x/2)(x\,dy)}{\int_0^1 (x\,dy)}$$

where $\tilde{x} = x/2$

Since $y = x^3$, $x = y^{1/3}$. Substituting into above for x:

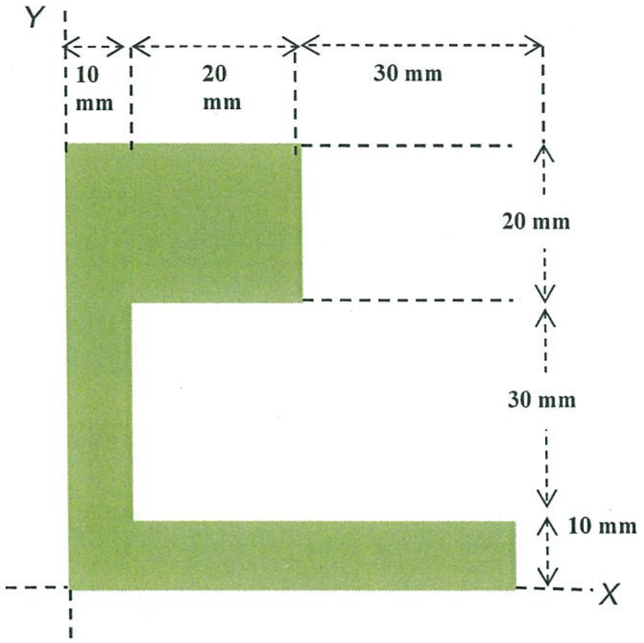
$$\bar{x} = \frac{\int_0^1 (y^{1/3}/2)\, y^{1/3}\, dy}{\int_0^1 y^{1/3}\, dy} = \frac{\frac{1}{2}\int_0^1 y^{2/3}\, dy}{\int_0^1 y^{1/3}\, dy} = \frac{\frac{3}{10}\left[y^{5/3}\right]_0^1}{\frac{3}{4}\left[y^{4/3}\right]_0^1} = \frac{3/10}{3/4} = \underline{0.4}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^1 y\,(x\,dy)}{\int_0^1 (x\,dy)} \qquad \text{where } \tilde{y} = y$$

$$\bar{y} = \frac{\int_0^1 y \cdot y^{1/3}\,dy}{\int_0^1 y^{1/3}\,dy} = \frac{\int_0^1 y^{4/3}\,dy}{\int_0^1 y^{1/3}\,dy} = \frac{\frac{3}{7}\left[y^{7/3}\right]_0^1}{\frac{3}{4}\left[y^{4/3}\right]_0^1} = \frac{3/7}{3/4} = 0.57$$

The coordinates of the centroid are (0.4, 0.57)

Question 2 Locate the centroid of the composite area shown below with respect to the *X*- and *Y*-axes. (Ivanoff, V. 2010. *Engineering Mechanics*, McGraw Hill)



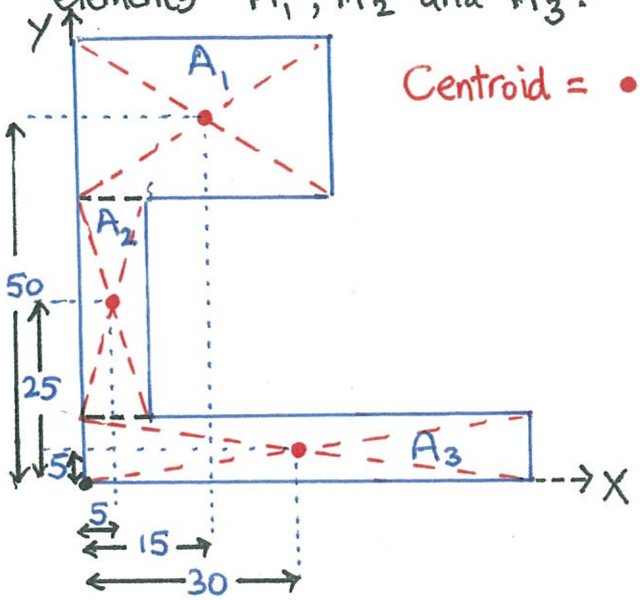
Centroid of a composite area

$$\bar{x} = \frac{\sum (Ax)}{\sum (A)} \qquad \bar{y} = \frac{\sum (Ay)}{\sum (A)}$$

Worked Solution 2

Divide area into 3 rectangular elements A_1, A_2 and A_3.

Centroid = •



where x = horizontal distance to centroids $A_1 A_2 A_3$ from Y-axis
y = vertical distance to centroids $A_1 A_2 A_3$ from X-axis

Element	Area	Distance		Area Moment	
	A	x	y	Ax	Ay
1	600	15	50	9 000	30 000
2	300	5	25	1 500	7 500
3	600	30	5	18 000	3 000
Σ =	1 500	-	-	28 500	40 500

$$\bar{x} = \frac{\Sigma(Ax)}{\Sigma(A)} = \frac{28500}{1500} = 19\text{mm} \qquad \bar{y} = \frac{\Sigma(Ax)}{\Sigma(A)} = \frac{40500}{1500} = 27\text{mm}$$

The coordinates of the centroid are (19mm, 27mm)