TR1.5: ANGULAR MEASUREMENT

Definition of a radian
So far all angles have been given in degrees. Another common unit of angular measurement is the radian.

If we move around the edge of a circle of radius = 1 (unit circle), a distance of 1 unit from A to P, in an anti-clockwise direction, then the angle \( AOP \) is formed.

\[ \text{The measure of this angle is } 1 \text{ radian} \left( \frac{\pi}{180} \right) \]

Converting between radians and degrees
The circumference of a circle is: \( 2 \times \pi \times \text{radius} \) units

Circumference of the unit (radius=1) circle is: \( 2 \times \pi \times 1 = 2\pi \) units.

Moving around the edge of the unit circle a distance of \( 2\pi \) units from A forms a complete circle.

The angle, in radians, formed by one full revolution of a circle is \( 2\pi \).

The angle, in degrees, of a circle is 360°.

\[ \therefore \quad 2\pi \degree = 360° \]

\[ \pi \degree = 180° \]

Rearranging the above statements gives:

\[ 1 \degree = \frac{180°}{\pi} \quad \text{and} \quad 1° = \frac{\pi}{180} \]

These two equations are used to convert between radians and degrees.
Examples

1. Convert 60° to radians.
   \[ 1^\circ = \frac{\pi^c}{180} \]
   \[ 60^\circ = \frac{\pi^c}{180} \times 60^c \]
   \[ 60^\circ = \frac{\pi^c}{3} \]
   Using \( \pi \approx 3.142 \), \( 60^\circ \approx 1.05^c \)

See Exercise 1

2. Convert 250° to radians.
   \[ 1^\circ = \frac{\pi^c}{180} \]
   \[ 250^\circ = \frac{\pi^c}{180} \times 250^c \]
   \[ 250^\circ = \frac{25\pi^c}{18} \]
   Using \( \pi \approx 3.142 \), \( 250^\circ \approx 4.36^c \)

See Exercise 2

Note: The symbol for radian, \( ^c \), is often omitted.

Exercise

1. Convert the following degrees to radians
   1. 30°
   2. 270°
   3. 20°
   4. 450°
   5. 135°
   6. 57.3°

2. Convert the following radians to degrees
   1. \( \frac{\pi}{2} \)
   2. \( \frac{5\pi}{4} \)
   3. \( \frac{11\pi}{6} \)
   4. 3.5π
   5. \( \pi \)
   6. 1

Answers

1. Degrees to radians
   1. \( \frac{\pi}{6} \)
   2. \( \frac{3\pi}{2} \)
   3. \( \frac{\pi}{9} \)
   4. \( \frac{5\pi}{2} \)
   5. \( \frac{3\pi}{4} \)
   6. 1

2. Radians to degrees
   1. 90°
   2. 225°
   3. 330°
   4. 630°
   5. 180°
   6. 57.3°