FU1.2: ABSOLUTE VALUE

The absolute value of a number $|x|$ gives a measure of its size or magnitude regardless of whether it is positive or negative. If a number is plotted on a number line then its absolute value can be considered to be the distance from zero.

Examples

Eg

(i) $|2| = 2$
(ii) $|-2| = 2$
(iii) $|-4 + 3| = |-1| = 1$
(iv) $|-8| + |-1| = 8 + 1 = 9$
(v) $|x| = 7 \Rightarrow x = 7$ or $x = -7$

The absolute value function and its graph

The absolute value function is a hybrid function defined as follows:

$f: \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

with graph

The domain of $f(x) = |x|$ is $\mathbb{R}$

The range is $\mathbb{R}^+ \cup 0$

The graph of $y = |x|$ may be translated in the same way as the graphs of other functions.

Compare the graphs of the following functions with that of $y = |x|

1. $y = |x-2|
2. $y = |x| + 1$
3. $y = 3 - |x| = -|x| + 3$
To sketch the graph of $y = \lfloor f(x) \rfloor$ we need to sketch the graph of $y = f(x)$ first and then reflect in the $x$-axis the portion of the graph which is below the $x$-axis.

**Example**

**Sketch** $\{(x, y): y = \lfloor x^2 - 1 \rfloor\}$

The graph of this function is the graph of $y = x^2 - 1$ with the portion below the $x$-axis reflected across the $x$-axis.

**Equations & inequalities involving $\lfloor f(x) \rfloor$**

Because $y = \lfloor f(x) \rfloor$ is a hybrid function two cases must be considered when solving equations and inequalities.

**Examples**

1. **Solve** $|x - 2| = 3$
2. If $|x - 2| = 3$
   then $x - 2 = 3$ or $-(x - 2) = 3$
   ie $x = 5$ or $x = -1$

With an absolute value expression on each side of the equation it is easier to square both sides:

2. **Solve** $|2x + 1| = |x - 5|$

If $|2x + 1| = |x - 5|$
then $(2x + 1)^2 = (x - 5)^2$
ie $4x^2 + 4x + 1 = x^2 - 10x + 25$
ie $3x^2 + 14x - 24 = 0$
ie $(3x - 4)(x + 6) = 0$
ie $x = \frac{4}{3}$ or $x = -6$

NB: Care must be taken when multiplying or dividing by a negative to reverse an inequality

3. **Solve** $\left| \frac{2 - x}{3} \right| < 4$
\[ \left| \frac{2-x}{3} \right| < 4 \quad \Rightarrow \quad \left| 2 - x \right| < 12 \]
\[ \Rightarrow -12 < 2 - x < 12 \]
\[ \Rightarrow -14 < -x < 10 \]
\[ \Rightarrow 14 > x > -10 \quad \text{or} \quad -10 < x < 14 \]

**Exercises**

**Exercise 1**
Evaluate
1. \( |-11| \)
2. \( |-9 + 4| \)
3. \( |-4| - |-5| \)
4. \( |-12| - |3| \)
5. \( |-30| + |5| \)

**Exercise 2**
Sketch the graph of
1. \( y = |x + 4| \)
2. \( y = |x - 1| - 3 \)
3. \( y = |3 - x^2| \)

**Exercise 3**
Find for \( x \in \mathbb{R} \)
1. \( \{ x: |x| = 6 \} \)
2. \( \{ x: |x - 1| < 3 \} \)
3. \( \{ x: \frac{|x - 3|}{2} \geq 1 \} \)
4. \( \{ x: \frac{|x|}{2} = |x + 2| \} \)

**Answers**

**Exercise 1**
1) 11 2) 5 3) -9 4) 9 5) 6

**Exercise 2**
1. \[
\begin{array}{c}
\text{1.} \\
\end{array}
\]
2. \[
\begin{array}{c}
\text{2.} \\
\end{array}
\]
3. \[
\begin{array}{c}
\text{3.} \\
\end{array}
\]

**Exercise 3**
1. \( \{-6,6\} \)
2. \( \{ x: -2 < x < 4 \} \)
3. \( \{ x: x \leq 1 \} \cup \{ x: x \geq 5 \} \)
4. \( \{ -4, -\frac{4}{3} \} \)