RMIT UNIVERSITY

A3.6 POLYNOMIAL LONG DIVISION

One polynomial may be divided by another of lower degree by long division (similar to arithmetic long division). For example:

$$\frac{(x^3+3x^2+x+9)}{(x+2)}$$

1. Write the question in long division form.

$$x+2\overline{)x^3+3x^2+x+9}$$

2. Begin with the x^3 term.
x^3 divided by x equals x^2.
Place x^2 above the division bracket as shown.

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{)x^3+3x^2+x+9} \\ \text{subtract} & \;\; x^3+2x^2 \\ & \qquad\quad x^2 \end{array}$$

3. Multiply $x+2$ by x^2.

$$x^2(x+2)=x^3+2x^2$$

Place x^3+2x^2 below x^3+3x^2 then subtract.

$$x^3+3x^2-(x^2+2x)=x^2$$

4. "Bring down" the next term (x) as indicated by the arrow.

$$\begin{array}{rl} & x^2+x \\ x+2 & \overline{)x^3+3x^2+x+9} \\ & \;\; x^3+2x^2 \;\downarrow \\ & \qquad\quad \underline{x^2+x} \end{array}$$

Repeat this process with the result until there are no terms remaining.

5. x^2 divided by x equals x.

Multiply:

$$x(x+2)=x^2+2x.$$

$$(x^2+x)-(x^2+2x)=-x$$

6. Bring down the 9.

7. $-x$ divided by x equals -1, then multiply

$$-1(x+2)=-x-2.$$

$$(-x+9)-(-x-2)=11$$

$$\begin{array}{rl} & x^2+x-1 \\ x+2 & \overline{)x^3+3x^2+x+9} \\ & \;\; x^3+2x^2 \;\downarrow\;\downarrow \\ & \qquad\quad \underline{x^2+x} \;\;\downarrow \\ & \qquad\quad x^2+2x \;\downarrow \\ & \qquad\qquad\; \underline{-x+9} \\ & \qquad\qquad\; -x-2 \\ & \qquad\qquad\qquad \underline{+11} \end{array}$$

The remainder is 11

$$\frac{(x^3+3x^2+x+9)}{(x+2)} \text{ equals } x^2+x-1\text{, with remainder 11.}$$

If the polynomial has an 'x^n' term missing, add the term with a coefficient of zero.

Example

$$\left(2x^3-3x+1\right)\div\left(x-1\right)$$

Rewrite $\left(2x^3-3x+1\right)$ as $\left(2x^3+0x^2-3x+1\right)$

Divide using the method from the previous example.

$2x^3 \div x = 2x^2$

$2x^2\left(x-1\right)=2x^3-2x^2$

$2x^3-0x^2-\left(2x^3-2x^2\right)=2x^2$

$2x^2 \div x = 2x$

$2x\left(x-1\right)=2x^2-2x$

$2x^2-3x-\left(2x^2-2x\right)=-x$

$-x \div x = -1$

$-1\left(x-1\right)=-x+1$

$-x+1-\left(x+1\right)=0$

Remainder is 0

$$\begin{array}{r|l} & 2x^2+2x-1 \\ \hline x-1 & 2x^3+0x^2-3x+1 \\ & 2x^3-2x^2 \quad\downarrow\quad\downarrow \\ & \underline{2x^2-3x} \quad\downarrow \\ & 2x^2-2x \quad\downarrow \\ & \underline{-x+1} \\ & -x+1 \\ & \underline{0} \end{array}$$

$$\left(2x^3-3x+1\right)\div\left(x-1\right)=\left(2x^2+2x-1\right) \text{ with remainder} = 0$$

$$\therefore\left(2x^3-3x+1\right)=\left(x-1\right)\left(2x^2+2x-1\right)$$

See Exercise 1.

Polynomial long division can be used help factorise cubic (and higher order) equations.

Solving Cubic Equations: The Factor Theorem

If P(x) is a polynomial in x and P(a) = 0 then (x – a) is a factor of P(x)

We can use the factor theorem to find one factor of a cubic function, and then use polynomial long division to find the remaining factor(s).

[Sometimes it is possible to find all solutions by finding three values of x for which P(x) = 0]

A cubic equation has a maximum of three distinct solutions.

Example

Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$

Look for a value of x that makes P(x) = 0.

Try $x = 1$ first, then work up through the factors of 24.

Try $x = 1$ P(1) =18
$x - 1$ is not a factor

Try $x = 2$ P(2) = 8
$x - 2$ is not a factor

Try $x = -2$ P(–2) = 0
P(–2) = 0 therefore
$x + 2$ is a factor.

Next divide $x^3 - 5x^2 - 2x + 24$ by ($x + 2$), using the method of polynomial long division shown above.

The remainder should be zero.

$$
\begin{array}{r}
x^2 - 7x + 12 \\
x + 2 \overline{\smash{)}\, x^3 - 5x^2 - 2x + 24} \\
x^3 + 2x^2 \qquad\qquad \\
\underline{-7x^2 - 2x} \qquad \\
-7x^2 - 14x \qquad \\
\underline{12x + 24} \\
12x + 24 \\
\underline{0}
\end{array}
$$

The factors of $x^3 - 5x^2 - 2x + 24$ are ($x + 2$) and ($x^2 - 7x + 12$)

The factors of $x^2 - 7x + 12$ are ($x - 3$) and ($x - 4$).

Therefore $x^3 - 5x^2 - 2x + 24 = 0$ has solutions x = – 2, x = 3 and x = 4

See Exercise 2.

Exercise 1

Divide the first polynomial by the second.

(a) $2x^3 - 6x^2 + 5x + 2, \quad x - 2$

(b) $3x^3 + 13x^2 + 6x - 12, \quad 3x + 4$,

(c) $3x^3 + x - 1, \quad x + 1$

(d) $x^3 + 2x - 3, \quad x - 1$

(e) $2x^4 + 5x^3 + x + 3, \quad 2x + 1$

(f) $2x^3 + x^2 + 5x + 12, \quad 2x + 3$

Exercise 2

Solve the following equations.

(a) $x^3 + 7x^2 + 11x + 5 = 0$

(b) $4x^3 + 2x^2 - 2x = 0$

(c) $-x^3 - 3x^2 + x + 3 = 0$

(d) $x^3 - 7x - 6 = 0$

Answers

Exercise 1

(a) $2x^2 - 2x + 1$ remainder 4

(b) $x^2 + 3x - 2$ remainder -4

(c) $3x^2 - 3x + 4$ remainder – 3

(d) $x^2 + x + 3$ remainder 0

(e) $x^3 + 2x^2 - x + 1$ remainder 2

(f) $x^2 - x + 4$ remainder 0

Exercise 2

(a) $x = -5, x = -1$

(b) $x = 0, x = 1, x = -1/2$

(c) $x = 1, \quad x = -3, \quad x = -1$

(d) $x = -1, x = -2, x = 3$