A3.3 FACTORISATION: DIFFERENCE OF TWO SQUARES

Difference of two squares (DOTS)

Consider the following expansion: $(x + 5)(x - 5) = x^2 + 5x - 5x - 25$

$= x^2 - 25$

Note that:

- the terms in the brackets differ only in the sign of the second term
- the expansion is the difference of two terms, both of which are perfect squares

In general: $\boxed{(a+b)(a-b)=a^2-b^2}$

This can be used to factorise expressions of the form $a^2 - b^2$

Examples

1. $a^2-36=(a)^2-(6)^2$ — expression is the **difference** of two squares

$a^2-36=(a+6)(a-6)$ — 'DOTS' rule (order of the terms on right hand side is not important)

2. $4x^2-y^2=(2x)^2-(y)^2$ — expression is the **difference** of two squares

$4x^2-y^2=(2x-y)(2x+y)$ — 'DOTS' rule

3. $3x^2-48=3(x^2-16)$ — **not the difference of two squares** but to factorise, take out a common factor of 3

$3x^2-48=3((x)^2-(4)^2)$

$3x^2-48=3(x+4)(x-4)$ — factorise using 'DOTS' rule

4. $(x+2)^2-9=(x+2)^2-3^2$ — expression is the **difference** of two squares

$(x+2)^2-9=((x+2)-3)((x+2)+3)$ — 'DOTS' rule

$(x+2)^2-9=(x-1)(x+5)$ — Simplify

5. y^2+36

this is **not the difference** of two squares. It is the **sum** of two squares.

No real factors

Exercise

Factorise the following using the 'DOTS' rule (if possible).

1. x^2-4
2. a^2-100
3. $49-x^2$
4. $64x^2-1$
5. $121x^2-49y^2$
6. a^2b^2-25
7. $5x^2-20$
8. $4a^2+100$
9. x^2y^3-36y
10. $(x+2)^2-y^2$
11. $(x-5)^2-36$
12. $(a+1)^2-(b-2)^2$

Answers

1. $(x-2)(x+2)$
2. $(a+10)(a-10)$
3. $(7+x)(7-x)$
4. $(8x-1)(8x+1)$
5. $(11x+7y)(11x-7y)$
6. $(ab-5)(ab+5)$
7. $5(x+2)(x-2)$
8. Does not factorise
9. $y(xy+6)(xy-6)$
10. $(x-y+2)(x+y+2)$
11. $(x-11)(x+1)$
12. $(a+b-1)(a-b+3)$