

## A3.2 FACTORISATION: PERFECT SQUARES

### Perfect squares

Examples of perfect squares are  $5^2$ ,  $x^2$ ,  $a^2 b^2$ ,  $(xy)^2$  and  $(a+b)^2$

$(a+b)^2$  can be expanded as follows

$$(a+b)^2 = a^2 + 2ab + b^2$$

similarly  $(a-b)^2 = a^2 - 2ab + b^2$

Note that:

- the expansion gives **three** terms

$$(a+b)^2 \text{ does not equal } a^2 + b^2$$

$$(a-b)^2 \text{ does not equal } a^2 - b^2$$

- the **first** and **last** terms of the expansion must be **positive**
- the **middle term** is **twice** the **product of the first and last terms**
- the middle term may be positive or negative

The rule for expanding perfect squares can be used in reverse for factorising perfect squares.

### Examples

Check each of the following expressions and, if it is a perfect square, state the perfect square.

1.  $x^2 + 14x + 49$

$$x^2 + 14x + 49 = (x+7)^2$$

first term is the square of  $x$

last term is positive and is the square of  $\pm 7$

middle term =  $2 \times x \times (+7)$

2.  $y^2 - 20y + 25$

$y^2 - 20y + 25$  is not a perfect square

first term is the square of  $y$

last term is positive and is the square of  $\pm 5$

middle term **does not equal**  $2 \times y \times (\pm 5)$

3.  $4a^2 - 12a - 9$

$4a^2 - 12a - 9$  is not a perfect square

first term is the square of  $2a$

last term is negative ( $9$  is a perfect square but  $-9$  is **not**)

4.  $100x^2 - 180x + 81$

$$100x^2 - 180x + 81 = (10x - 9)^2$$

first term is  $(10x)^2$

last term is  $(\pm 9)^2$

middle term is  $2 \times 10x \times (-9)$

5.  $50x^2 + 80x + 32 = 2(25x^2 + 40x + 16)$

$$50x^2 + 80x + 32 = 2(5x + 4)^2$$

neither the first or last term is a perfect square therefore the expression cannot be a perfect square, but to factorise the expression:

take out a common factor of 2.

factorise  $25x^2 + 40x + 16$  which is a perfect square.

### Exercise

Check each of the following expressions. If it is a perfect square state the perfect square.

1.  $a^2 + 2a + 1$

2.  $x^2 - 4x + 4$

3.  $25x^2 - 10x + 1$

4.  $4y^2 - 6y + 9$

5.  $81x^2 + 108x + 36$

6.  $9a^2 - 24a - 16$

7.  $16x^2 - 40xy + 25y^2$

8.  $121z^2 + 88z + 64$

9.  $2x^2 + 8x + 8$

### Answers

1.  $(a + 1)^2$

2.  $(x - 2)^2$

3.  $(5x - 1)^2$

4. Not a perfect square

5.  $(9x + 6)^2$

6. Not a perfect square

7.  $(4x - 5y)^2$

8. Not a perfect square

9. Not a perfect square but

$$2x^2 + 8x + 8 = 2(x^2 + 4x + 4) = 2(x + 2)^2$$