

A3.1 FACTORISATION: COMMON FACTORS

Expansion of brackets (or removing brackets) in an algebraic expression is done by multiplying all the terms inside the brackets by the term(s) outside the brackets.

Example $5a(3y - 2x) = 15ay - 10ax$

Each term inside the brackets has been multiplied by $5a$.

Factorisation is the reverse of expansion. To factorise a number or algebraic expression means to write the number or expression as a product (multiplication).

Examples

1. $-2xyz$ has factors -2 , x , y , and z
2. $7(a + b)$ has factors 7 and $(a + b)$
3. $3(x - y)$ has factors 3 and $(x - y)$
4. $x(3a + 2b)$ has factors x and $(3a + 2b)$
5. $2p(2r + 1)(s + 4)$ has factors 2 , p , $(2r + 1)$ and $(s + 4)$

Expansion means removing brackets

Factorisation means inserting brackets

Factorisation by removing a common factor

- Search each term in the expression for a common factor (**every term** must have this factor)
- There may be several common factors. Search until you have found all of them
- If there is more than one common factor multiply them to give highest common factor. (HCF)
- The HCF is placed before the bracket
- The terms inside the bracket are found by dividing each term by the HCF.

Examples

1. $5y + 10$
 $= 5 \times y + 5 \times 2$ *5 times y equals 5y and 5 times 2 equals 10*
 $= 5(y + 2)$ *factorise, common factor 5*

2. $3x + 3y$
 $= 3 \times x + 3 \times y$
 $= 3(x + y)$ *common factor 3*

3. $p^2 + p$
 $= p \times p + p \times 1$
 $= p(p + 1)$ *common factor p*

4. $7y^2 + 7y$
 $= 7y \times y + 7y \times 1$
 $= 7y(y + 1)$ *common factors 7 and y HCF=7y*

5. $2abc - 12ac$
 $= 2 \times a \times c \times b - 2 \times a \times c \times 6$
 $= 2ac(b - 6)$ *common factor 2, a, and c. HCF = 2ac*

See Exercise 1

A common factor may be negative.

Further Examples (negative factors)

6. $-2a - 2b$
 $= (-2) \times a + (-2) \times b$
 $= -2(a + b)$ *common factor -2*

7. $-3x + 6xy$
 $= (-3x) \times 1 - (-3x) \times 2y$
 $= -3x(1 - 2y)$ *HCF = -3x*

See Exercise 2

Exercise 1.

Factorise the following (if possible)

- | | | |
|-------------------------|------------------------|---------------------------|
| a) $3x + 3y$ | b) $2a - 2b$ | c) $8a - 8b + 8c$ |
| d) $xy - 5x$ | e) $x^2 - x$ | f) $7x + 21y$ |
| g) $5m - 2n$ | h) $c^2 - 2bc - 3c$ | i) $5mn - 10n$ |
| j) $3m^2 - 3mnp$ | k) $7x + 21x^2$ | l) $12m^2 - 18mn$ |
| m) $5xy - 10xz$ | n) $5pq - pq^2 - 3pqr$ | o) $2ab^2c + 6abc^2$ |
| p) $rst + 5rst^2 - 2rs$ | q) $5mn + 10m - pqr$ | r) $5xyz - x^2yz^2 + 10x$ |

Exercise 2

Factorise the following by removing a negative factor.

- | | | |
|------------------|----------------------------|------------------|
| a) $-3x - 6y$ | b) $-15xy + 25xz$ | c) $-2xy + 4xyz$ |
| d) $14xyz - 7xy$ | e) $-6xyz - 15yz - 3xy^2z$ | |

Answers

Exercise 1

- | | | |
|-----------------------|---------------------|--------------------------|
| a) $3(x + y)$ | b) $2(a - b)$ | c) $8(a - b + c)$ |
| d) $x(y - 5)$ | e) $x(x - 1)$ | f) $7(x + 3y)$ |
| g) no factors | h) $c(c - 2b - 3)$ | i) $5n(m - 2)$ |
| j) $3m(m - np)$ | k) $7x(1 + 3x)$ | l) $6m(2m - 3n)$ |
| m) $5x(y - 2z)$ | n) $pq(5 - q - 3r)$ | o) $2abc(b + 3c)$ |
| p) $rs(t + 5t^2 - 2)$ | q) no factors | r) $x(5yz - xyz^2 + 10)$ |

Exercise 2

- | | | |
|--------------------|------------------------|-------------------|
| a) $-3(x + 2y)$ | b) $-5x(3y - 5z)$ | c) $-2xy(1 - 2z)$ |
| d) $-7xy(-2z + 1)$ | e) $-3yz(2x + 5 + xy)$ | |