RMIT UNIVERSITY

A2.3 TRANSPOSITION WITH CHALLENGES

When trying to transpose formulae, there are a number of possible complications.

1. The subject variable appears more than once in the formula!

For example: Make 'm' the subject of $E = m g h + ½ m v^2$

Suggested procedure: **Move all terms containing 'm' to one side of the formula and factorize.**

Examples

(i) $E = m g h + ½ m v^2$ [All 'm's are already on one side of the formula]

$E = m (g h + ½ v^2)$ [Factorize]

$\frac{E}{gh+\frac{1}{2}v^2} = m$ [÷ (gh + ½v²)]

or $m = \frac{E}{gh+\frac{1}{2}v^2}$

(ii) $I r = E - I R$

$I r + I R = E$ [move 'I's to one side of ormula]

$I (r + R) = E$ [factorize]

$I = \frac{E}{r+R}$ [÷ (r + R)]

2. The formula contains lots of fractions!!

For example: Make 'u' the subject of $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Suggested procedure: **Cross multiply if possible (ie if there is only one fraction on each side of equation) otherwise eliminate fractions by multiplying both sides by the lowest common denominator.**

Examples

(i) *Express in terms of 'u'* $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ [LCD is fuv]

$fuv . \frac{1}{f} = fuv. \frac{1}{v} - fuv. \frac{1}{u}$ [multiply all terms by LCD (fuv)]

$$u v = f u - f v$$ [cancel and simplify]

$$u v - f u = -f v$$ [move all terms containing 'u' to one side]

$$u (v - f) = -f v$$ [factorize]

$$u = \frac{-fv}{v - f}$$ [÷ (v – f)]

(ii) *Transpose* $m = \sqrt{\dfrac{d-s}{s(e-f)}}$ *to make 's' the subject.*

$$m = \sqrt{\frac{d - s}{s(e - f)}}$$ [The radical sign acts as a bracket and is removed first]

$$m^2 = \frac{d - s}{s(e - f)}$$ [Square both sides]

$$m^2s(e-f)=1(d-s)$$ [cross multiply $(m^2 = \dfrac{m^2}{1})$]

$$m^2se - m^2sf = d - s$$ [simplify]

$$m^2se - m^2sf + s = d$$ [shift all terms with 's' to one side]

$$s(m^2e - m^2f + 1) = d$$ [factorize]

$$s = \frac{d}{m^2e - m^2f + 1}$$ [÷ $(m^2e - m^2f + 1)$]

3. *The subject is an exponent!!!*

For example: Make 't' the subject of $Q = A \times 10^{kt}$

Suggested procedure: **Use logarithms - Remember** $\mathbf{\log x^n = n \log x}$

Examples:

(i) *Express in terms of 't'* $Q = A \times 10^{kt}$

then $\dfrac{Q}{A} = 10^{kt}$ [÷ 'A' both s]

$$\log\left(\frac{Q}{A}\right) = \log 10^{kt}$$ [take logs both sides]

$$\log\left(\frac{Q}{A}\right) = kt \log 10$$ [$\log x^n = n\log x$]

$$\log\left(\frac{Q}{A}\right) = kt$$ [$\log 10 = 1$]

$$\frac{\log\left(\frac{Q}{A}\right)}{k} = t \qquad [\div \text{ k both sides}]$$

or $$t = \frac{1}{k}\log\left(\frac{Q}{A}\right)$$

(ii) *Make 'n' the subject of the formula* $S = P(1+i)^n$

$$S = P(1+i)^n$$

$$\frac{S}{P} = (1+i)^n \qquad [\div \text{ P both sides}]$$

$$\log\left(\frac{S}{P}\right) = \log(1+i)^n \qquad [\text{take logs both sides}]$$

$$\log\left(\frac{S}{P}\right) = n\log(1+i) \qquad [\log x^n = n\log x]$$

$$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1+i)} \qquad [\div \log(1+i)]$$

Sometimes it is helpful to change a logarithmic equation into its equivalent exponential form:

$$y = \log_a x \iff a^y = x$$

(iii) *Transform the formula* $T = \frac{1}{c}\log_e(m-A)$ *to make 'm' the subject*

$$T = \frac{1}{c}\log_e(m-A)$$

$$cT = \log_e(m-A) \qquad [\times \text{c both sides}]$$

$$e^{cT} = m - A \qquad [\text{change to exponential equation}]$$

$$m = e^{cT} + A \qquad [+ \text{A both sides}]$$

Exercises

Transpose the following formulae to make the variable in brackets the subject.

1. $M = 10.5C + 35.2(W - \frac{C}{8})$ [C]

2. $At = M(P + t)$ [t]

3. $I = \frac{E}{R}$ [R]

4. $\frac{P}{Q} = \frac{R}{S}$ [S]

5. $I = \frac{E}{R+r}$ [r]

6. $W = \frac{2PR}{R-r}$ [R]

7. $A = \sqrt{\frac{2q(L-r)}{rL}}$ [L]

8. $E = \frac{w^2 a}{(w^2 + m)b^3}$ [w]

9. $H = Ae^{-kt}$ [t]

10. $\frac{1}{q^2}\log_e\left(\frac{M}{2}\right) = P$ [M]

Answers (NB: there may be alternative answers that are algebraically equivalent)

1. $C = \frac{M - 35.2W}{6.1}$

2. $t = \frac{MP}{A-M}$

3. $R = \frac{E}{I}$

4. $S = \frac{RQ}{P}$

5. $r = \frac{E - IR}{I}$

6. $R = \frac{wr}{w-2P}$

7. $L = \frac{2qr}{2q - A^2 r}$

8. $w = \pm\sqrt{\frac{Eb^3 m}{a - Eb^3}}$

9. $-\frac{1}{k}\log_e\left(\frac{H}{A}\right)$

10. $M = 2e^{pq^2}$