A2.1 REARRANGING FORMULAE

Some of the most important equations that we might be required to transpose occur frequently in science, engineering and economics. They are called formulae and give a general rule describing the relationship between variable quantities.

Here are some examples: $A = \pi r^2$

$s = ut + \frac{1}{2}at^2$

$S = P(1+i)^n$

In these examples A, s and S are, respectively, the subjects of the formulae. Sometimes a formula is given in an inconvenient form and it is necessary to rearrange the formula to make a different variable the subject

We know $A = \pi r^2$ but $r = ???$

and $s = ut + \frac{1}{2}at^2$ but $a = ???$

Before beginning to rearrange these more complex formulae, let us examine the processes required to make 'A' the subject in each of the following simple equations.

We may perform whichever operations we choose providing we do the same to each side of the equation.

(i) $A + B = C$

$A + B - B = C - B$ [- B both sides]

$\therefore A = C - B$

(ii) $A - B = C$

$A - B + B = C + B$ [+ B both sides]

$\therefore A = C + B$

(iii) $AB = C$

$\frac{AB}{B} = \frac{C}{B}$ [÷ B both sides]

$\therefore A = \frac{C}{B}$

(iv) $\frac{A}{B} = C$

$\frac{A}{B} \times B = C \times B$ [x B both sides]

$\therefore \quad A = C \times B$

(v) $A^2 = B$

$\sqrt{A^2} = \sqrt{B}$ [$\sqrt{}$ both sides]

$\therefore \quad A = \sqrt{B}$

(vi) $\sqrt{A} = B$

$\left(\sqrt{A}\right)^2 = B^2$ [square both sides]

$\therefore \quad A = B^2$

Notice that $-$ 'undoes' $+$ and $+$ 'undoes' $-$

$\div$ 'undoes' $\times$ and $\times$ 'undoes' $\div$

$\sqrt{x}$ 'undoes' x^2 and x^2 'undoes' $\sqrt{x}$

or more generally, $\sqrt[n]{x}$ 'undoes' x^n and x^n 'undoes' $\sqrt[n]{x}$.

This is because each of these pairs of operations are *inverse* operations.

Examples

1. *Transform $V = A - K$ to make 'A' the subject*

$V = A - K$

$V + K = A - K + K$ [add k both sides]

$V + K = A$ or $A = V + K$

2. *Make 'd' the subject of $C = \pi \times d$*

$C = \pi \times d$

$\frac{C}{\pi} = \frac{\pi \times d}{\pi}$ [divide π both sides]

$\frac{C}{\pi} = d$ or $d = \frac{C}{\pi}$

3. *Rearrange $j = 3w - 5$ in terms of 'w'.*

$j = 3w - 5$

$j + 5 = 3w - 5 + 5$ [add 5 both sides]

$j + 5 = 3w$

$$\frac{j+5}{3} = \frac{3w}{3} \qquad \text{[divide 3 both sides]}$$

$$\frac{j+5}{3} = \text{w or w} = \frac{j+5}{3}$$

4. *Make 'c' the subject of $E = mc^2$*

$$E = mc^2$$

$$\frac{E}{m} = c^2 \qquad [\div \text{ m both sides}]$$

$$\sqrt{\frac{E}{m}} = c \qquad [\sqrt{\ } \text{both sides}]$$

Exercises

1. $m = n - 2$ Find n

2. $A = 2B + C$ Find C

3. $A = 2B + C$ Find B

4. $P = \frac{k}{v}$ Find k

5. $PV = k$ Find V

6. $v = u + at$ Find a

7. $v = u + at$ Find t

8. $r = \sqrt{\frac{A}{\pi}}$ Find A

9. $A = x^2$ Find x

10. $A = \pi r^2$ Find r

Answers

1. $n = m + 2$ **2.** $C = A - 2B$ **3.** $B = \frac{A-C}{2}$ **4.** $k = PV$ **5.** $V = \frac{k}{P}$

6. $a = \frac{v-u}{t}$ **7.** $t = \frac{v-u}{a}$ **8.** $A = \pi r^2$ **9.** $x = \pm\sqrt{A}$ **10.** $r = \pm\sqrt{\frac{A}{\pi}}$