

A1.6 ALGEBRAIC FRACTIONS

(QUADRATICS)

More complicated algebraic fractions may contain quadratic expressions or fractions within fractions.

Examples:

$$\begin{aligned}
 1. \quad \frac{2}{x^2 + x - 12} - \frac{1}{x^2 - 9} &= \frac{2}{(x+4)(x-3)} - \frac{1}{(x-3)(x+3)} \quad [\text{factorize to see the factors in the denominator}] \\
 &= \frac{2(x+3)}{(x+4)(x-3)(x+3)} - \frac{(x+4)}{(x+4)(x-3)(x+3)}
 \end{aligned}$$

[we must have a common denominator before we add fractions and the common denominator must contain one of every factor]

$$\begin{aligned}
 &= \frac{2(x+3) - (x+4)}{(x+4)(x-3)(x+3)} \\
 &= \frac{x+2}{(x+4)(x-3)(x+3)}
 \end{aligned}$$

$$2. \quad \text{Simplify } \frac{1 - \frac{2}{x}}{1 + \frac{2}{x}}$$

$$\begin{aligned}
 \frac{1 - \frac{2}{x}}{1 + \frac{2}{x}} &= \left(1 - \frac{2}{x}\right) \div \left(1 + \frac{2}{x}\right) \\
 &= \left(\frac{x}{x} - \frac{2}{x}\right) \div \left(\frac{x}{x} + \frac{2}{x}\right) \\
 &= \frac{x-2}{x} \div \frac{x+2}{x} \\
 &= \frac{x-2}{x} \times \frac{x}{x+2} \\
 &= \frac{x-2}{x+2}, \quad x \neq 0
 \end{aligned}$$

3. Simplify $\frac{\frac{-3}{x^2+2x-3} + \frac{1}{x-1}}{\frac{3}{x-1} + 3}$

$$\frac{\frac{-3}{x^2+2x-3} + \frac{1}{x-1}}{\frac{3}{x-1} + 3} = \left(\frac{-3}{x^2+2x-3} + \frac{1}{x-1} \right) \div \left(\frac{3}{x-1} + 3 \right)$$

$$= \left(\frac{-3}{(x+3)(x-1)} + \frac{x+3}{(x+3)(x-1)} \right) \div \left(\frac{3}{x-1} + \frac{3(x-1)}{x-1} \right)$$

$$= \frac{-3 + x + 3}{(x+3)(x-1)} \div \frac{3 + 3x - 3}{x-1}$$

$$= \frac{x}{(x+3)(x-1)} \div \frac{3x}{x-1}$$

$$= \frac{x}{(x+3)(x-1)} \times \frac{x-1}{3x}$$

$$= \frac{1}{3(x+3)} \quad x \neq 1, x \neq 0$$

Exercise

1. $\frac{2}{x^2+2x} + \frac{1}{x^2-4}$

2. $\frac{1-\frac{6}{x}}{\frac{x}{2}-3}$

3. $\frac{\frac{1}{x^2-4} + \frac{1}{2x+4}}{1 + \frac{2}{x-2}}$

Answers

1. $\frac{3x-4}{x(x+2)(x-2)}$

2. $\frac{2}{x}, x \neq 6$

3. $\frac{1}{2(x+2)}, x \neq 0, x \neq 2$