

A1.5 ALGEBRAIC FRACTIONS

(MULTIPLICATION & DIVISION)

Simplifying fractions

Remember $\frac{18}{24} = \frac{\cancel{1}8^3}{\cancel{2}4^4} = \frac{3}{4}$ because 18 and 24 have a common factor of 6.

$\frac{5}{20} = \frac{\cancel{5}^1}{\cancel{2}0^4} = \frac{1}{4}$ because 5 and 20 have a common factor of 5.

Algebraic fractions may be simplified in a similar way by cancelling *factors* that are common to the numerator and denominator.

Examples

$$1 \quad \frac{4x^2y}{6y^2} = \frac{\cancel{4}^2xy}{\cancel{6}^3yy} = \frac{2x^2}{3y}$$

$$2 \quad \frac{a(b+2c)}{2ab} = \frac{\cancel{a}(b+2c)}{2\cancel{a}b} = \frac{b+2c}{2b}$$

$$3 \quad \frac{m-n}{(m-n)^2} = \frac{m-n}{(m-n)(m-n)} = \frac{1}{m-n} \quad [(m-n) \text{ in the numerator cancels with } (m-n) \text{ in the denominator}]$$

$$4 \quad \frac{3x^2y}{6x+9y} = \frac{\cancel{3}x^2y}{\cancel{3}(2x+3y)} = \frac{x^2y}{2x+3y} \quad [\textit{Factorising helps you to see factors!}]$$

$$5 \quad \frac{p-2}{6p-3p^2} = \frac{p-2}{3p(2-p)} = \frac{p-2}{-3p(p-2)} = -\frac{1}{3p} \quad [\textit{it is very useful to know that } 2-p = -(p-2)]$$

NB: Only *factors* may be cancelled

eg. $\frac{x+2}{2y} \neq \frac{x+1}{y}$ because 2 is NOT a *factor* of $x+2$

See *Exercise 1*

Multiplication of Fractions

Remember $\frac{18^3}{81} \times \frac{24^3}{387} = \frac{9}{7}$

Similarly $\frac{5a^1}{71} \times \frac{14^2}{a1} = 10$ Any factor in the numerator can be cancelled with any factor in the denominator

Examples

1. $\frac{x}{6(x-2)} \times \frac{3(x-2)}{x^2} = \frac{1}{2x}$ [after cancelling]

2. $\frac{3m+12}{10} \times \frac{5}{m^2+4m} = \frac{3(m+4)}{10} \times \frac{5}{m(m+4)}$ [factorise first]
 $= \frac{3}{2m}$ [after cancelling]

Division of Fractions

Remember $\frac{5}{4} \div \frac{19}{8} = \frac{5}{4} \times \frac{8}{19}$ [change to multiply and invert the second fraction]
 $= \frac{5}{41} \times \frac{8^2}{19}$ [cancel]
 $= \frac{10}{19}$

Division with algebraic fractions is very similar

- *Invert and multiply*
- *Factorise (if necessary) and cancel*
- *Simplify*

Examples

1. $\frac{7p}{12} \div \frac{3}{8} = \frac{7p}{12} \times \frac{8}{3}$
 $= \frac{7p}{12^3} \times \frac{8^2}{3}$
 $= \frac{14p}{9}$

$$\begin{aligned}
 2. \quad \frac{m^2}{n} \div 6m &= \frac{m^2}{n} \div \frac{6m}{1} \\
 &= \frac{m^2}{n} \times \frac{1}{6m} \\
 &= \frac{m^1 \times m}{n} \times \frac{1}{6m^1} \\
 &= \frac{m}{6n} \quad \text{[after cancelling]}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{4(x+3)}{9} \div \frac{24}{5x} &= \frac{4^1(x+3)}{9} \times \frac{5x}{24_6} \\
 &= \frac{5x(x+3)}{54}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{2a+4}{15} \div \frac{a+2}{6} &= \frac{2a+4}{15} \times \frac{6}{a+2} \\
 &= \frac{2(a+2)^1}{\cancel{15}_5} \times \frac{\cancel{6}^2}{(a+2)_1} \\
 &= \frac{4}{5}
 \end{aligned}$$

See *Exercise 2*

Exercise 1

Simplify the following fractions

1 $\frac{12ab^2}{8bc}$

2 $\frac{5x-20}{5}$

3 $\frac{9u-18}{2u-4}$

4 $\frac{6t-9}{12-8t}$ HINT: $3-2t = -(2t-3)$

5 $\frac{b}{b^2+7b}$

6 $\frac{(j+4)(j-4)}{3j+12}$

7 $\frac{2(5-v)}{3v-15}$

8 $\frac{9r^2-3r}{16r-48r^2}$

Exercise 2

1 Simplify

a $\frac{4}{5} \times \frac{15}{16}$

b $\frac{4a}{3} \times \frac{9}{a}$

c $\frac{32h^2}{9j} \times \frac{27j}{48h}$

d $\frac{3d-2}{3} \times \frac{4}{3d-2}$

e $\frac{2r+4}{3r-9} \times \frac{5r-15}{7r+14}$

f $\frac{10p-5}{3} \times \frac{3q+3}{2p-1}$

g $\frac{4g^2-6g}{8} \times \frac{3}{6g-9}$

h $\frac{3-2y}{33y-11} \times \frac{18y^2-6y}{7-2y}$

2 Simplify

a $\frac{4m-16}{m} \div \frac{8m-32}{8m}$

b $\frac{6xy-5y^2}{4x+10y} \div \frac{12x^2-10xy}{12x+30y}$

Answers

Exercise 1

1 $\frac{3ab}{2c}$

2 $x-4$

3 $\frac{9}{2}$

4 $-\frac{3}{4}$

5 $\frac{1}{b+7}$

6 $\frac{j-4}{3}$

7 $-\frac{2}{3}$

8 $-\frac{3}{16}$

Exercise 2

1 a $\frac{3}{4}$

b 12

c 2h

d $\frac{4}{3}$

e $\frac{10}{21}$

f $5(q+1)$

g $\frac{g}{4}$

h $\frac{6y(3-2y)}{11(7-2y)}$

2 a 4

b $\frac{3y}{2x}$