

# A1.3 REMOVING BRACKETS

The *distributive law* enables us to expand brackets like  $a(b + c)$  ie  $a \times (b + c)$

$$\begin{aligned} a \times (b + c) &= a \times b + a \times c \\ \text{or } a(b+c) &= ab + ac \end{aligned}$$

$$\text{so } 2 \times (3 + 5) = 2 \times 3 + 2 \times 5$$

## Examples

$$\begin{aligned} 1. \quad 3(x + 2) &= 3 \times x + 3 \times 2 \\ &= 3x + 6 \end{aligned}$$

$$\begin{aligned} 2. \quad -5(m + 4) &= (-5) \times m + (-5) \times 4 \\ &= -5m + (-20) \\ &= -5m - 20 \end{aligned}$$

$$3. \quad -2(p - 7) = -2p + 14$$

$$4. \quad e(e + 2) = e^2 + 2e \quad [\text{leaving out the second step}]$$

$$\begin{aligned} 5. \quad -(4p - 3) &= (-1)(4p - 3) \quad [\text{nb: } -x = (-1)x] \\ &= -4p + 3 \end{aligned}$$

$$\begin{aligned} 6. \quad 4(x - 3) + 2(x + 1) &= 4x - 12 + 2x + 2 \\ &= 6x - 10 \quad [\text{after simplifying}] \end{aligned}$$

$$\begin{aligned} 7. \quad 6k(2k + 3) - 3k(k - 3) &= 12k^2 + 18k - 3k^2 + 9k \\ &= 9k^2 + 27k \end{aligned}$$

*See Exercise 1*

## Binomial Products

A *binomial product* looks like this:  $(m + 2)(m + 3)$ .

It is the product of two *binomial* (two term) expressions. To expand the two brackets, multiply each term in one bracket by each term in the other bracket and simplify if possible.

$$\begin{aligned}(m + 2)(m + 3) &= m(m + 3) + 2(m + 3) \\ &= m \times m + 3 \times m + 2 \times m + 2 \times 3 \\ &= m^2 + 3m + 2m + 6 \\ &= m^2 + 5m + 6\end{aligned}$$

These steps can be shown algebraically as:

$$(A + B)(C + D) = AC + AD + BC + BD$$

Important special cases of the expansion are:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) & (a - b)^2 &= (a - b)(a - b) \\ &= a^2 + ab + ab + b^2 & &= a^2 - ab - ab + b^2 \\ &= a^2 + 2ab + b^2 & &= a^2 - 2ab + b^2\end{aligned}$$

### Examples

$$\begin{aligned}1. (2f + 3)(g + 5h) &= 2f \times g + 3 \times g + 2f \times 5h + 3 \times 5h \\ &= 2fg + 3g + 10fh + 15h \quad [\text{sometimes it is not possible to simplify further}]\end{aligned}$$

$$\begin{aligned}2. (p - 2)(p - 7) &= p^2 - 2p - 7p + 14 \\ &= p^2 - 9p + 14\end{aligned}$$

$$\begin{aligned}3. (2w + 3)(w - 4) &= 2w^2 + 3w - 8w - 12 \\ &= 2w^2 - 5w - 12\end{aligned}$$

$$\begin{aligned}4. (4r - 3s)^2 &= (4r - 3s)(4r - 3s) \\ &= 16r^2 - 12rs - 12rs + 9s^2 \\ &= 16r^2 - 24rs + 9s^2\end{aligned}$$

**See Exercise 2**

## Exercise 1

### 1. Remove the brackets

- a)  $2(x - 5)$                       b)  $3(6 - b)$                       c)  $(b + 5)c$   
d)  $2x(2a + 3b)$                       e)  $-6(p - 2q + 4)$                       f)  $3y(yz - 2y + 1)$

### 2. Expand and simplify

- a)  $3(x + 1) + 2(x + 2)$                       b)  $2(p - 1) - (p - 3)$                       c)  $2(4m - 3) - 5(9 - 2m)$   
d)  $2w(3w + 1) + 3w(2w - 5)$                       e)  $2a(a - 1) - a(a - 3)$                       f)  $5j(3j + 2) - 2j(4j - 1)$   
g)  $3q(5q + 4) + 6(8 - 3q)$                       h)  $2r(3r - 1) - 3r(3 + 2r)$

3 Expand the following expressions and simplify where possible remembering the correct order of operations.

- a)  $4(2m - 3) + 8$                       b)  $9 - 3(4b + 3)$                       c)  $1 - 4(x - 1)$   
d)  $4 - (5 - 2x)$                       e)  $8m - 3(1 - 2m) + 6$                       f)  $9c(4 - c) + 2(c - 7)$

## Exercise 2

### 1. Expand the following binomial products

- a)  $(x + 5)(x + 3)$                       b)  $(u - 5)(u + 3)$                       c)  $(k + 6)(j + 1)$   
d)  $(y + 2)(y - 2)$                       e)  $(e - 7)(e - 8)$                       f)  $(t - 3)(5 - t)$   
g)  $(3q + 2)(q + 1)$                       h)  $(8a + 5)(a - 3)$                       i)  $(2c - d)(3c + 2d)$   
j)  $(3a - 2b)(3a + 2b)$

### 2. Remove the brackets

- a)  $(d + 3)^2$                       b)  $(x - 2)^2$                       c)  $(3v + 2)^2$   
d)  $(y - 3z)^2$                       e)  $(3 - 2q)^2$                       f)  $(4f - 3gh)^2$

## Answers

### Exercise 1

- 1 a)  $2x - 10$     b)  $18 - 3b$     c)  $bc + 5c$     d)  $4ax + 6bx$     e)  $-6p + 12q - 24$     f)  $3y^2z - 6y^2 + 3y$   
2 a)  $5x + 7$     b)  $p + 1$     c)  $18m - 51$     d)  $12w^2 - 13w$     e)  $a^2 + a$     f)  $7j^2 + 12j$   
g)  $15q^2 - 6q + 48$     h)  $-11r$   
3 a)  $8m - 4$     b)  $-12b$     c)  $5 - 4x$     d)  $-1 + 2x$     e)  $14m + 3$     f)  $38c - 9c^2 - 14$

### Exercise 2

- 1 a)  $x^2 + 8x + 15$     b)  $u^2 - 2u - 15$     c)  $kj + 6j + k + 6$     d)  $y^2 - 4$     e)  $e^2 - 15e + 56$   
f)  $-t^2 - 15 + 8t$     g)  $3q^2 + 5q + 2$     h)  $8a^2 - 19a - 15$     i)  $6c^2 + cd - 2d^2$     j)  $9a^2 - 4b^2$   
2 a)  $d^2 + 6d + 9$     b)  $x^2 - 4x + 4$     c)  $9v^2 + 12v + 4$     d)  $y^2 - 6yz + 9z^2$     e)  $9 - 12q + 4q^2$   
f)  $16f^2 - 24fgh + 9g^2h^2$