A1.1 ALGEBRAIC OPERATIONS

Like Terms

Like terms contain exactly the same *pronumerals* (letters, variables).

Like Terms	Unlike terms
3x, 5x	3x, 4y
2a, -3a	3a, 3
$3m^2$, m^2	$3m^2$, 3m
2ab, 3ab	$2a^2b$, $3ab^2$
3ef, 5fe	3ef, 7fg

NB: Order is unimportant but alphabetical order of pronumerals is conventional

See Exercise 1

Addition and Subtraction

Only like terms may be added or subtracted.

1. $7e + 10e = (7 + 10)e$
   $= 17e$ [eventually it is possible to just THINK the second step and go straight to the answer]

2. $3x^2 - x^2 - 4x^2 = (3 - 1 - 4)x^2$
   $= -2x^2$

3. $3m - 4n + 6m + n = (3 + 6)m + (-4 + 1)n$
   $= 9m - 3n$

4. $3a - b - 5a + 4ab - 3b + ab = (3 - 5)a + (-1-3)b + (4 + 1)\,ab$
   $= -2a - 4b + 5ab$

5. $3x - x^2$ cannot be simplified

6. $p + 2p - 3 = 3p - 3$

7. $8uv + 3u - 10vu = -2uv + 3u$

8. $6r^2s - 2rs^2$ cannot be simplified

See Exercise 2

Multiplication

Consider the sign (positive or negative) of the answers to the following simple multiplication problems

$2 \times 3 = 6$	*positive* × *positive* → *positive*
$-2 \times 3 = -6$	*negative* × *positive* → *negative*
$2 \times -3 = -6$	*positive* × *negative* → *negative*
$-2 \times -3 = 6$	*negative* × *negative* → *positive*

Terms can be multiplied whether they are like or unlike terms.

When multiplying two or more terms consider:

- the sign of the answer
- the product of the numbers

and use an index to show

- how many factors of each pronumeral

1. $(-4) \times (-3b) = 12b$
2. $-2 \times 6y = -12y$
3. $2e \times (-5e^2) = -10e^3$
4. $(-2u^2v) \times (-4v) = 8u^2v^2$
5. $-3pq \times (-2q) \times p = 6p^2q^2$

In algebra a fraction line means to divide eg. ½ = 1 ÷ 2, ¾ = 3 ÷ 4

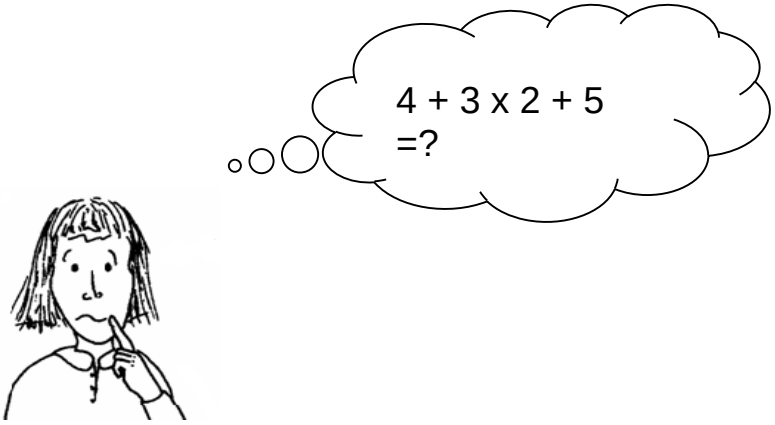
When dividing algebraic expressions:

- rewrite as a fraction if necessary
- expand any powers
- establish the sign of the answer
- cancel factors
- write the answer in simplified form

1. $-12wyz \div 3yz = \dfrac{-12wyz}{3yz} = -4w$
2. $-2m^2n \div 6mn^2 = \dfrac{-2m^2n}{6mn^2} = \dfrac{-2mmn}{6mnn} = -\dfrac{m}{3n}$
3. $6a^2 \times 4b \div 12ab = \dfrac{6aax4b}{12ab} = 2a$

NB: $\dfrac{m+2}{4m}$ **CANNOT** be simplified!!!

See Exercise 3



Order of Operations

Consider $4 + 3 \times 2 + 5$. ***What is the answer?***

$4 + 3 \times 2 + 5 \rightarrow 7 \times 7 = 49?$

$4 + 3 \times 2 + 5 \rightarrow 4 + 3 \times 7 = 25?$

$4 + 3 \times 2 + 5 \rightarrow 7 \times 2 + 5 = 19?$

To avoid such confusion we perform operations in the following order

1. **B**rackets	**B**
2. **I**ndices	**I**
3. **M**ultiplication and **D**ivision from left to right	**M**
	D
4. **A**ddition and **S**ubtraction from left to right	**A**
	S

To make it easier to remember the rule for order of operations is abbreviated to **B I M D A S**

Using this rule $4 + 3 \times 2 + 5 = 4 + 6 + 5 = 15$!!

Examples:

1. $3 \times 2 + 4 = 6 + 4 = 10$
2. $3 + 2 \times 4 = 3 + 8 = 11$
3. $(3 + 2) \times 4 = 5 \times 4 = 20$
4. $3 - 2^2 = 3 - 4 = -1$
5. $(3 - 2)^2 = 1^2 = 1$
6. $3^2 - 2^2 = 3 \times 3 - 2 \times 2 = 9 - 4 = 5$
7. $3st - 3s \times 4t = 3st - 12st = -9st$

See Exercise 4

Exercise 1

Which of the five terms on the right is a like term with the term on the left?

1) $3x$ 3 2x $3x^2$ 2xy $4x^2a$

2) $2ab$ 2a 2b $3x^2$ 6abc 12ab

3) $2x^2$ x 2x $5x^2$ 4x $4x^3$

4) $3xy^2$ 3 3xy $3x^2$ xy^2 $3y^2$

5) $2m^2n$ n mn^2 8mn $2m^2$ $4nm^2$

6) $4ab^2c$ 4 $2ab^2c$ 4abc $8b^2c$ 4cba

Exercise 2

Simplify each of the following.

1. a) $5x + 3x$ b) $12x - 7x$
   c) $15xy + 5xy$ d) $11mn - 5mn$
   e) $10abc - 3bca$ f) $10m - 22m$
   g) $6x + 3x + 4x$ h) $4ab + 5ab - 2ab$
   i) $m + 2m - 9m$ j) $14xy - 4xy + 2xy$

2. a) $13x + 4 - 3x - 1$ b) $10mn + 5m + 12mn + 6m$
   c) $3xy^2 + 2xy + 5xy^2 + 3xy$ d) $5xy + 6m - 2xy - 2m$
   e) $x + y + 2x - y$ f) $3a + 5b - a - 6b$
   g) $x + 4y - x - 2y$ h) $7x - 4m - 5x - 3m$
   i) $4x - 5x - 3y + 5x$ j) $9mn - 3m - n + 4m^2$

Exercise 3

Simplify the following algebraic expressions

1. a) $5 \times 2k$ b) $4a \times 3ab$
   c) $y \times 3y$ d) $4m \times (-3mn)$
   e) $m \times 3p \times 5$ f) $2ab \times 3bc \times (-4)$
   g) $2ab^2 \times 3ac$ h) $4m \times (-5kmp)$

2. a) $18ef \div 6f$ b) $-100uvw \div 100w$
   c) $24gh^2 \div 8gh$ d) $3m^2 n \div 12mn^2$
   e) $rs \times 2st \div 2s$ f) $3jk \times 12km \div 9jkm$
   g) $10p \times 3pq \div 16pq$ h) $4yz \times 5w^2z \div 10wy$

Exercise 4

Simplify

1) $18 + (3 \times -5)$
2) $3 \times -4 + (8 \times 2)$
3) $10 - 5^2 + 3$
4) $(10 - 5)^2 + 3$
5) $10^2 - 5^2$
6) $(10 - 5)^2$
7) $3m + 2 \times 3m$
8) $6ab - 3a \times 4b$
9) $16gh - 4gh \times 4$
10) $9b - 3b \times 2k + 2k \times b$

Answers

Exercise 1

1. 2x 2. 12ab 3. $5x^2$ 4. xy^2 5. $4nm^2$ 6. $2ab^2c$

Exercise 2

1. a. 8x b. 5x c. 20xy d. 6mn e. 7abc f. -12m g. 13x h. 7ab i. -6m j. 12xy
2. a. $10x + 3$ b. $22mn + 11m$ c. $8xy^2 + 5xy$ d. $3xy + 4m$ e. 3x f. $2a - b$ g. 2y
h. $2x - 7m$ i. $4x - 3y$ j. $9mn - 3m - n + 4m^2$

Exercise 3

1. a. 10k b. $12a^2b$ c. $3y^2$ d. $-12m^2n$ e. 15mp f. $-24ab^2c$ g. $6a^2b^2c$ h. $-20km^2p$
2. a. 3e b. -uv c. 3h d. $\frac{m}{4n}$ e. rst f. 4k g. $\frac{15p}{8}$ h. $2wz^2$

Exercise 4

1. 3 2. 4 3. -12 4. 28 5. 75
6. 25 7. 9m 8. -6ab 9. 0 10. 9b -4bk