SAMPLE SPACES

A list or diagram showing all possible outcomes in a probability experiment is called a sample space.

\[
\text{Then } \Pr(E) = \frac{\text{number of ways } E \text{ can occur}}{\text{number of outcomes in the sample space}} = \frac{n(E)}{n(S)}
\]

- For tossing a single die, the sample space is 1, 2, 3, 4, 5, 6
  and \( \Pr(1) = \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{6} \)

- For this spinner, which has 4 equal sectors, the sample space is Red, Green, Yellow, Blue
  And \( \Pr(R) = \Pr(G) = \Pr(Y) = \Pr(B) = \frac{1}{4} \)

NB: The sum of the probabilities of the distinct outcomes within a sample space is 1.

Tree diagrams

A tree diagram can be used to find the sample space.

For example, if two coins are tossed there are four possible outcomes:

The sample space for tossing two coins is HH, HT, TH, TT

If \( E \) is the event 'at least one head' then \( \Pr(E) = \Pr(\text{HH or HT or TH}) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \)

The sample space for a three child family is shown below:

If \( E \) is the event 'first child a girl' then \( \Pr(E) = \frac{4}{8} = \frac{1}{2} \)
Other sample spaces and diagrams

Sample Space for Choosing a Card from a Deck

If a single card is drawn from the deck and

(a) \( D \) is the event ‘the card is a diamond’ then \( \Pr(D) = \frac{13}{52} = \frac{1}{4} \)

(b) \( E \) is the event ‘the card is a diamond (D) or an ace (A)’

then \( \Pr(E) = \Pr(D \text{ or } A) \)

\[ = \Pr(D \cup A) \]

\[ = \Pr(D) + \Pr(A) - \Pr(D \cap A) \]

\[ = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} \]

\[ = \frac{16}{52} \]

\[ = \frac{4}{13} \]

Tables and Venn diagrams can also be used to organise information that makes finding probabilities easier

The diagram shows the number of people in a survey of 256 who regularly ate Kit Kats, Mars Bars or Rocky Road

From the diagram we can see

\[ \Pr(K) = \frac{26+46+30+38}{256} = \frac{140}{256} = \frac{35}{64} \]

\[ \Pr(M \cap R) = \frac{78}{256} = \frac{39}{128} \]

\[ \Pr(\text{KitKat and MarsBar but not Rocky Road}) = \frac{30}{256} = \frac{15}{128} \]

\[ \Pr(\text{at least one of these things}) = 1 - \frac{20}{256} = \frac{236}{256} = \frac{59}{64} \] [using complementary events]
From the table we can see
\[
\Pr(S) = \frac{100}{200} = \frac{1}{2}
\]
\[
\Pr(C') = \frac{160}{200} = \frac{4}{5}
\]
\[
\Pr(S \cap C) = \frac{30}{200} = \frac{3}{20}
\]

Exercise
1. Use a tree diagram to find the sample space for a two child family. Hence find
   (a) The probability that both children are girls
   (b) The probability that the oldest child is a girl
   (c) The probability that at least one child is a girl

2. The diagram shows the sample space for tossing a single die twice.

<table>
<thead>
<tr>
<th>First throw</th>
<th>Second throw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1)</td>
</tr>
<tr>
<td>2</td>
<td>(2,1)</td>
</tr>
<tr>
<td>3</td>
<td>(3,1)</td>
</tr>
<tr>
<td>4</td>
<td>(4,1)</td>
</tr>
<tr>
<td>5</td>
<td>(5,1)</td>
</tr>
<tr>
<td>6</td>
<td>(6,1)</td>
</tr>
</tbody>
</table>

Find the probability that
   (a) the first toss is a 4
   (b) the sum of the two tosses is 5
   (c) at least one toss is a 6
   (d) neither toss is a 6

3. In a classroom of 20 Yr 12 VCE students 10 study Maths Methods, 7 study Specialist maths and 5 study both. Organise the information in a Venn diagram and find the probability that a student chosen at random
   (a) Studies neither of these maths subjects
   (b) Studies Maths Methods but not Specialist Maths

4. Find the probability that a card drawn at random from a pack is
   (a) A red card
   (b) Lower than a 5 (ace low)

Answers
1. (a) \(\frac{1}{4}\) (b) \(\frac{1}{2}\) (c) \(\frac{3}{4}\)
2. (a) \(\frac{1}{6}\) (b) \(\frac{1}{9}\) (c) \(\frac{11}{36}\) (d) \(\frac{25}{36}\)
3. (a) \(\frac{2}{5}\) (b) \(\frac{1}{4}\)
4. (a) \(\frac{1}{2}\) (b) \(\frac{4}{13}\)